Supply Chain Self-Control

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Abstract

In our model, each firm in a supply network faces a self-control-based trade-off between investing in stronger relationships with their suppliers and saving costs in the short term. A firm wants to have robust and reliable supply relationships in the long-term but also wants to minimize the expenses in the present. This could lead to under-investment in relationship strength as well as fragility in the supply network for strictly psychological reasons. A possible commitment device for this problem could be a contract or an agreement that specifies the minimum level of investment or quality that each firm must provide to the suppliers or customers. This could create incentives and penalties for maintaining strong relationships and avoiding disruptions in the supply network. Alternatively a commitment device could be a third-party intermediary or platform that monitors and enforces the quality and reliability of the supply relationships.

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1 Introduction

"The wood from which it [the pencil] is made, for all I know, comes from a tree that was cut down in the state of Washington. To cut down that tree, it took a saw. To make the saw, it took steel. To make steel, it took iron ore. This black center—we call it lead but it's really graphite, compressed graphite—I'm not sure where it comes from, but I think it comes from some mines in South America. This red top up here, this eraser, a bit of rubber, probably comes from Malaya, where the rubber tree isn't even native! It was imported from South America by some businessmen with the help of the British government.

–Milton Friedman, "Free to Choose," (pilot), (1980).

"I know of no other piece of literature that so succinctly, persuasively, and effectively illustrates the meaning of both Adam Smith's invisible hand—the possibility of cooperation without coercion—and Friedrich Hayek's emphasis on the importance of dispersed knowledge and the role of the price system in communicating information that will make the individuals do the desirable things without anyone having to tell them what to do."

–Milton Friedman, on Leonard Read's (1958) *I, Pencil*, quoted in LeClaire (2023).

Governments and multinational firms alike have incentives to sustain and strengthen global supply chains[1]. Anticipating this in his day, Milton Friedman used a pencil as a powerful example of how complex supply chains work and how they illustrate the power of voluntary cooperation in networks. He also believed that this is an example of how the market coordinates the actions of millions of people through prices and profits, without any central planning. It seems that Friedman was against centralized policy planning, but not necessarily against *psychological* planning in the sense of individual self-control and foresight. In fact, he might have supported the idea of a farsighted planner and myopic doer as a way of explaining why firms in networks sometimes fail to act in their own long-term interest. However, he would probably argue for market-based solutions.

This overall conceptualization seems in line with what is a behavioral operations revolution in supply chain operations research. Behavioral operations is a multi-disciplinary branch of operations management that considers the effects of human behavior in process performance, influenced by cognitive biases (e.g. Sterman, 1988).[2] What remains is integrating these descriptive discussions with the conveniently established behavioral economics revolution. Although the above papers have made the argument for behavioral foundations of supply chains, there is a need for a technical architecture to formalize it.

This paper attempts to meet this need. In this paper, I build a novel class of supply network models with self-control problems to propose a unified psychology-and-economic theory of brittle

[1]For example, an iPhone may contain parts from China, Taiwan, Japan, South Korea, Germany, France, and the United States. These parts are then assembled in factories in China or India, and shipped to retailers and consumers globally. Similarly, a typical car may have over 30,000 parts that are manufactured by hundreds or thousands of suppliers. These parts include engines, transmissions, batteries, tires, electronics, and more. The parts are then assembled in plants that may be located in different countries or continents, depending on the brand and model of the car. The finished cars are then transported by trucks, trains, ships, or planes to dealers and customers. Clothing is another product category that has complex supply chains. The raw materials for clothing, such as cotton, wool, silk, or synthetic fibers, may be grown or produced in different countries. These materials are then processed into fabrics and dyed in mills that may be located elsewhere. The fabrics are then cut and sewn into garments by factories that may be in different regions or countries. The garments are then shipped to warehouses, distributors, and retailers before reaching the end consumers.

[2]Behavioral operations research is the study of attributes of human behavior and cognition that impact the design, management, and improvement of operating systems. See for e.g., Schweitzer and Cachon (2000), Bendoly et al., 2006; Loch an Wu, 2007; Größler et al., 2008; Bendoly et al., 2010; Knemeyer and Naylor, 2011, Croson et al., 2013).

delivery graphs that reconciles the behavioral operations literature with the economic networks literature. In response to psychological and emotional shocks, the firms interact and their supply network relationships suffer. However, newer commitment devices can accommodates psychological shortcomings, and foster win-win scenarios. In so doing, I extend Laibson (1997) to a complex network environment. To provide a framework, I make significant modifications and extensions to account for the differences in the individual versus network environment.

Famously, a self-control or time-inconsistency problem is a situation where an individual or a group faces a conflict between their short-term desires and their long-term goals. For example, a person may want to eat a piece of cake now, but also want to lose weight in the future. A self-control problem can lead to suboptimal choices and outcomes, such as overeating, overspending, procrastination, or addiction. One possible way to overcome a self-control problem is to use a commitment device. A commitment device is a strategy or a tool that helps an individual or a group stick to their long-term goals by imposing costs or constraints on their short-term behavior. For example, a person may sign up for a gym membership, set up automatic savings, or use an app that blocks distracting websites.

The contribution of the paper is to think of supply chains as suffering from self-control problems, where we could imagine that each firm in the supply network faces a *psychological* trade-off between investing in stronger relationships with their suppliers in the long-term and saving costs in the short term. A firm may want to have robust and reliable supply relationships in the long term, but also want to minimize their expenses in the present. This leads us to a distinct but familiar framework to motivate under-investment in relationship strength and, as a consequence, fragility in the supply network. I also introduce novel algorithms to implement commitment devices for contracts and platforms.

I first show that firms in a supply network tend to invest too little in making their relationships stronger and more reliable, which makes the network weak and prone to failures. This is because each firm wants to save costs now rather than invest for the future. This theorem is important for supply chain management because it shows that there is a problem in how firms cooperate and coordinate in a network, and that there is a need for solutions that can make them invest more in their relationships.

Also, due to the network environment and its trascendence over the archetypical behavioral context, there are further additions that I am able to showcase. In the second part of the paper, I am able to motivate commitment devices designed to minimize the impact of self-control problems in the network. A possible commitment device for this problem could be a contract or an agreement that specifies the minimum level of investment or quality that each firm has to provide to their suppliers or customers. This could create incentives and penalties for maintaining strong relationships and avoiding disruptions in the supply network. Alternatively, a commitment device could be a third-party intermediary or a platform that monitors and enforces the quality and reliability of the supply relationships.

The first commitment device is a relevant contract. In supply chain management, the role of Contract and Supplier Management staff is to minimize the cost of ownership and maximize efficiencies in the supply chain (see The Procurement Journey (2023) for an illustration). Such staff assist with tasks such as: ensuring the contract is successfully executed. This responsibility includes meeting all special conditions relating to the performance of the contract which may cover economic, innovation-related, environmental, social or employment-related conditions. Contract and Supplier Management staff also ensure continuous improvement in projects. However, stakeholders such as Parley Pro (2023) note that "the importance of contract management is often pushed aside by procurement companies."[3], which is consistent with the self-control problems described in the preceding section.

I find that firms in a supply network can sign contracts with their partners that specify how much they have to invest in their relationships, and that punish them if they do not do so. It also find that there is a contract that can make the network stronger and less likely to fail. This is because a contract can create incentives and penalties for investing in relationships. By signing a contract, a firm promises to invest a certain amount and expects the same from its partner. This reduces the risk of failures and increases the future revenue for both firms. This theorem is important for supply chain management because it shows that there is a possible solution to the problem of under-investment in relationships: a contract that can enforce cooperation and coordination in a network. It also shows that there is a need for designing optimal contracts that can benefit both parties and overcome the challenges involved.

Finally, another type of commitment device may be a third-party intermediary or platform. The may be cloud-based or other technological platforms such as those based on blockchain technologies. The discussion is relevant for related platforms in general. Such innovative business models include cloud-based organization, digital platforms, service-oriented value creation, and dynamic process composition. For example, Amazon offers usage of its physical and digital assets to third-parties in the Fulfillment by Amazon business model (Amazon 2022). Another example is Google Cloud Platform. Similarly, Siemens and BMW developed smart manufacturing platforms using cloud technology (see Ivanov et al (2022) for an overview).

The objective of the relevant theorem is to state how firms in a supply network can use such a device or an intermediary or a platform that monitors and enforces their relationships, and that rewards or punishes them for complying or not[4]. It also says that there is a device or an intermediary or a platform that can make the network stronger and less likely to fail. This is because one of these mechanisms can create incentives and penalties for investing in relationships. By using one of these mechanisms, a firm promises to invest a certain amount and expects the same from its partner. This reduces the risk of failures and increases the future revenue for both firms. This theorem is important for supply chain management because it shows that there is another possible solution to the problem of under-investment in relationships, which is one of these mechanisms that can monitor and enforce cooperation and coordination in a network. It also shows that there is a need for choosing an appropriate mechanism that can benefit both parties and overcome the challenges involved.

The paper transcends a specific and stylized rational network model that captures only some aspects of the production process and the supply relationships, and which may not necessarily be realistic or representative of other settings or domains. The three theorems are more relevant to the supply chain management system context than the state-of-the-art in network economics because they provide more psychologically, general and applicable models and frameworks that can explain and predict the behavior and outcomes of firms in a supply network, and that can also inform and guide the design and implementation of interventions or mechanisms that can improve the performance and resilience of the supply network.

The psychology-and-economics orientation means that this paper thus marks a departure from recent work in economics. In the modern literature on supply chains in economics (see,in particular, the models in Costinot, Vogel and Wang (2013), Elliott, Golub and Leduc (2022) and surveys by Elliott and Golub (2022)), equilibrium supply chains are broadly seen as the manifestations of rational or optimal behavior assumed by traditional models and only vulnerable to external shocks[5].

[3] See Parley Pro (2023) at https://parleypro.com/blog/contract-negotiation-in-supply-chain-management/

[4] For a comprehensive overview of cloud supply chain platforms, see Ivanov et al (2022).

[5] This is an excellent approximation of the impact of shocks like the global lock downs from 2020 to 2022 more so

This paper provides a partial theoretical backing for recent empirical work that suggests that when a firm joins a supply chain, its sales may fluctuate somewhat in the short-term (Alfaro-Ureña, Manelici and Vasquez, 2022). It also helps unpack how supply chain members exposed to a large and exogenous decline in bank financing may pass this liquidity shock to their downstream customers (Costello, 2020). The commitment devices are novel.

The paper proceeds as follows. I first prove that self-control problems exist within the context of a complex industrial network. I then show how a special contract can lessen these shortcomings in the environment. Afterwards, I show that a platform can achieve the same end. I then close the paper.

2 Basic Model and Assumptions

I provide the basic mathematical notation and assumptions needed now. Let N be the set of all firms in the supply network, and let $n = |N|$ be the number of firms.

Let I_{ij} denote the level of investment or quality that firm i provides to firm j, where $i, j \in N$ and $i \neq j$. Let $\mathbf{I} = (I_{ij})_{i,j \in N}$ denote the matrix of investment or quality levels for all pairs of firms.

Let $c(I)$ denote the cost function of investment or quality for any firm, where c is an increasing and convex function. Let $c'(I)$ and $c''(I)$ denote the first and second derivatives of c with respect to I.

Let $\pi_{ij}(I_{ij})$ denote the probability of failure of relationship ij, where π_{ij} is a decreasing function of I_{ij}. Let $\pi'_{ij}(I_{ij})$ and $\pi''_{ij}(I_{ij})$ denote the first and second derivatives of π_{ij} with respect to I_{ij}.

Let $Y_i(\mathbf{I})$ denote the output of firm i, which is equal to 1 if firm i produces its output successfully, and 0 otherwise. The output of firm i depends on whether it receives its essential inputs from its suppliers, and whether it delivers its output to its customers. That is,

$$Y_i(\mathbf{I}) = \prod_{j \in S_i} (1 - \pi_{ji}(I_{ji})) \prod_{k \in C_i} (1 - \pi_{ik}(I_{ik})),$$

where S_i is the set of suppliers of firm i, and C_i is the set of customers of firm i.

- Let R denote the market revenue, which is shared equally among all firms that produce their output successfully. That is,

$$R = \frac{Z(\mathbf{I})}{\sum_i Y_i(\mathbf{I})},$$

where $Z(\mathbf{I})$ is the aggregate output of the supply network, which is equal to

$$Z(\mathbf{I}) = \sum_i Y_i(\mathbf{I}).$$

- Let $\beta \in (0, 1)$ denote the discount factor for each firm, which reflects its impatience or uncertainty about the future. - Let $\delta \in (0, 1)$ denote the long-run discount factor for each firm, which reflects its degree of commitment or loyalty to its partners.

The notation and assumptions are meant to capture the essential features of a supply network, where firms produce complex goods that require inputs from other firms, and where the quality and reliability of the inputs and outputs depend on the level of investment or quality that each firm provides to its partners.

than normal times of normal chain functioning in the absence of such shocks.

The cost function $c(I)$ reflects the trade-off that each firm faces between investing in stronger relationships and saving costs in the short term. The higher the level of investment or quality, the higher the cost for the firm. The convexity of the cost function implies that the marginal cost of investment or quality increases as the level of investment or quality increases.

The probability of failure function $\pi_{ij}(I_{ij})$ reflects the risk that each firm faces due to disruptions in its supply relationships. The lower the level of investment or quality that a firm provides to its partner, the higher the probability that their relationship will fail. The failure of a relationship means that one or both firms will not be able to produce their output due to a breakdown in their input or output delivery. The decreasing function π_{ij} implies that the marginal benefit of investment or quality decreases as the level of investment or quality increases.

The output function $Y_i(\mathbf{I})$ reflects the complementarities between inputs and outputs in the production process. The output of a firm depends on whether it receives its essential inputs from its suppliers, and whether it delivers its output to its customers. If any of these conditions is not met, then the firm will not be able to produce its output. The product form of Y_i implies that the output of a firm is zero if any of its relationships fails.

The market revenue R reflects the demand for the final product that is produced by the supply network. The market revenue is shared equally among all firms that produce their output successfully. This implies that each firm has an incentive to cooperate with its partners and invest in relationship strength, as this increases its expected revenue.

The discount factor β reflects the impatience or uncertainty of each firm about the future. The lower the discount factor, the less weight each firm puts on its future profits relative to its current profits. This implies that each firm has an incentive to save costs in the short term rather than invest in relationship strength, as this increases its current profit.

The long-run discount factor δ reflects the degree of commitment or loyalty of each firm to its partners. The lower the long-run discount factor, the less weight each firm puts on its long-run profits relative to its short-run profits. This implies that each firm has an incentive to switch partners rather than invest in relationship strength, as this increases its short-run profit.

2.1 Explaining the time-inconsistent preferences in the graph

Let $G = (N, E)$ be a directed graph that represents the supply network, where N is the set of nodes (firms) and E is the set of edges (relationships). Each edge $e = (i, j)$ has a weight $w_e = I_{ij}$, which denotes the level of investment or quality that firm i provides to firm j at time t. The matrix $\mathbf{I}_t = (I_{ij})_{i,j \in N}$ is the adjacency matrix of the graph G at time t. - Let $U_t(i)$ denote the utility function of node i at time t, which depends on its investment level I_{it}. The utility function is equal to the expected discounted profit of node i, minus the cost of investment. That is,

$$U_t(i) = \beta^t \mathbb{E}[\pi_t(i)] - c(I_{it}),$$

where $\beta \in (0, 1)$ is the discount factor, $\mathbb{E}[\pi_t(i)]$ is the expected profit of node i at time t, and $c(I_{it})$ is the cost of investment.

The utility function based on time-inconsistency as described assumes that node i has a present bias, meaning that it discounts its future profits more heavily than its current profits. This implies that the discount factor β is not constant, but depends on the time horizon of node i. That is,

$$\beta = \begin{cases} 1 & \text{if } t = 0, \\ \delta & \text{if } t > 0, \end{cases}$$

where $\delta \in (0, 1)$ is the long-run discount factor.

- This means that node i values its current profit more than its future profit, and that it may change its investment decision over time. For example, at time $t = 0$, node i may plan to invest a high level of I_{i0} to increase its future profit, but at time $t = 1$, it may revise its plan and invest a lower level of I_{i1} to save costs. This is because node i's discount factor changes from $\beta = 1$ to $\beta = \delta < 1$, which makes the future profit less attractive relative to the current cost.

Therefore, the utility function based on time-inconsistency as described can capture the dynamic inconsistency or self-control problem that node i may face when making investment decisions over time.

3 Results

I assume that each firm in the supply network has a hyperbolic discount function, which implies dynamically inconsistent preferences and a motive to constrain future choices. The firm has access to an imperfect commitment technology: an illiquid asset whose sale must be initiated one period before the sale proceeds are received. The firm's investment in relationship strength can be seen as a form of illiquid asset that reduces the risk of disruption in the future.

The first theorem makes explicit the idea that firms face a psychological trade-off between investing in stronger relationships and saving costs in the short term, and that this leads to underinvestment in relationship strength and fragility in the supply network. This is stated as follows:

Theorem 1. *Suppose that each firm in the supply network has a hyperbolic discount function with discount factor $\beta \in (0, 1)$ and long-run discount factor $\delta \in (0, 1)$.*

Suppose also that each firm has access to an illiquid asset whose sale must be initiated one period before the sale proceeds are received.

Let I_{ij} denote the investment that firm i makes in relationship strength with firm j, and let $c(I_{ij})$ denote the cost function of investment, which is increasing and convex. Let p_{ij} denote the probability that relationship ij fails, which is decreasing in I_{ij}. Let Y_i denote the output of firm i, which depends on whether it receives all its essential inputs from its suppliers. Let R_i denote the revenue of firm i, which depends on its output and its prices.

Then, there exists an equilibrium level of investment $\hat{I}_{ij}$ for each relationship ij such that:

- *(1) In each period, each firm maximizes its current utility subject to its budget constraint and its commitment constraint.*
- *(2) In equilibrium, each firm's consumption equals its income minus its investment cost.*
- *(3) In equilibrium, each firm's investment satisfies $\beta\delta E[R'_i|I_{ij} = \hat{I}_{ij}] = c'(\hat{I}_{ij})$, where R'_i is the next-period revenue of firm i.*
- *(4) In equilibrium, there is underinvestment in relationship strength relative to the socially optimal level, i.e., $\hat{I}_{ij} < I^*_{ij}$ for all ij, where I^*_{ij} satisfies $\delta E[R'_i|I_{ij} = I^*_{ij}] = c'(I^*_{ij}) + \sum_k \delta E[R'_k|I_{kj} = I^*_{kj}]$, where k ranges over all firms that are directly or indirectly affected by relationship ij.*
- *(5) In equilibrium, there is fragility in the supply network, i.e., there exists a threshold level of relationship strength $\bar{I}_{ij}$ such that if $\hat{I}_{ij} < \bar{I}_{ij}$ for any ij, then a small negative shock to relationship strength leads to a large discontinuous drop in aggregate output.*

Tying everything together, the first theorem states that each firm in the supply network faces a trade-off between investing in stronger relationships and saving costs in the short term, and that this leads to under-investment in relationship strength and fragility in the supply network. This means that each firm has an incentive to provide a low level of investment or quality to its partners, which increases the risk of disruption and reduces the expected revenue for both parties. The fragility of the supply network means that it is vulnerable to small negative shocks that can propagate and amplify throughout the network, causing large-scale failures and losses. This theorem is relevant to the supply chain management system context because it shows that there is a market failure in the provision of relationship strength, which is a public good that benefits all firms in the network. It also shows that there is a need for interventions or mechanisms that can align the interests of all firms and induce them to invest more in relationship strength than they would otherwise.

The complete proofs are in the Appendix. I sketch out a general explanation, giving an outline of the main steps and ideas. To prove part 1, we need to show that in each period, each firm maximizes its current utility subject to its budget constraint and its commitment constraint. The budget constraint says that the firm's consumption plus its investment cost cannot exceed its income. The commitment constraint says that the firm cannot sell its illiquid asset in the current period, but it can initiate a sale that will be completed in the next period. The firm's current utility depends on its consumption and its expected future utility, which is discounted by a factor β. The firm's expected future utility depends on its next-period revenue, which is uncertain and depends on whether its supply relationships fail or not. The firm chooses its consumption and its investment to maximize its current utility subject to the constraints.

For part 2, we need to show that in equilibrium, each firm's consumption equals its income minus its investment cost. This follows from the fact that the firm's utility function is strictly increasing and concave in consumption, and that the firm faces a binding budget constraint. Therefore, the firm will consume as much as possible given its income and investment cost.

To prove part 3, we need to show that in equilibrium, each firm's investment satisfies $\beta\delta E[R_i'|I_{ij} = \hat{I}_{ij}] = c'(\hat{I}_{ij})$, where R_i' is the next-period revenue of firm i. This follows from the first-order condition for the firm's optimization problem. The left-hand side of the equation represents the marginal benefit of investing more in relationship strength, which is equal to the discounted expected increase in next-period revenue due to a lower probability of failure. The right-hand side of the equation represents the marginal cost of investing more in relationship strength, which is equal to the derivative of the cost function. In equilibrium, the marginal benefit and the marginal cost are equalized.

For part 4, we need to show that in equilibrium, there is under-investment in relationship strength relative to the socially optimal level, i.e., $\hat{I}_{ij} < I_{ij}^*$ for all ij, where I_{ij}^* satisfies $\delta E[R_i'|I_{ij} = I_{ij}^*] = c'(I_{ij}^*) + \sum_k \delta E[R_k'|I_{kj} = I_{kj}^*]$, where k ranges over all firms that are directly or indirectly affected by relationship ij. This follows from a comparison of the equilibrium condition and the social optimum condition. The social optimum condition takes into account not only the expected increase in next-period revenue for firm i, but also for all other firms that are connected to firm i in the supply network. Therefore, the social optimum condition implies a higher marginal benefit of investing in relationship strength than the equilibrium condition. Since the cost function is increasing and convex, this implies that the socially optimal level of investment is higher than the equilibrium level of investment.

To prove part 5, we need to show that in equilibrium, there is fragility in the supply network, i.e., there exists a threshold level of relationship strength $\bar{I}_{ij}$ such that if $\hat{I}_{ij} < \bar{I}_{ij}$ for any ij, then a small negative shock to relationship strength leads to a large discontinuous drop in aggregate output. This follows from the fact that aggregate output is discontinuous in relationship strength due to complementarities between inputs. If relationship strength falls below a critical level for any pair of firms, then there is a positive probability that one or both firms will fail to produce their output due to a disruption in their supply relationship. This failure will propagate through the supply network and affect other firms that depend on their output as inputs. Therefore, a small negative shock to relationship strength can trigger a cascade of failures and a large drop in aggregate output.

3.1 The reasons for inefficiency of the equilibrium investment

To explain the logic of Theorem 1 with graph notation, it says that each node in the graph $G = (N, E)$ faces a trade-off between investing in stronger edges and saving costs in the short term, and that this leads to underinvestment in edge strength and fragility in the graph G. This means that each node has an incentive to provide a low weight to its adjacent edges, which increases the probability of edge failure and reduces the expected revenue for both nodes. The fragility of the graph G means that it is vulnerable to small negative shocks that can affect one or more edges, and that these shocks can propagate and amplify throughout the graph G, causing many nodes to fail and lose their revenue.

To see this, let $G = (N, E)$ be a directed graph that represents the supply network, where N is the set of nodes (firms) and E is the set of edges (relationships). Each edge $e = (i, j)$ has a weight $w_e = I_{ij}$, which denotes the level of investment or quality that node i provides to node j at time t. The matrix $\mathbf{I}_t = (I_{ij})_{i,j\in N}$ is the adjacency matrix of the graph G at time t.

I am letting $U_t(i)$ denote the utility function of node i at time t, which depends on its investment level I_{it}. The utility function is equal to the expected discounted profit of node i, minus the cost of investment. That is,

$$U_t(i) = \beta^t \mathbb{E}[\pi_t(i)] - c(I_{it}),$$

where $\beta \in (0, 1)$ is the discount factor, $\mathbb{E}[\pi_t(i)]$ is the expected profit of node i at time t, and $c(I_{it})$ is the cost of investment.

The utility function based on time-inconsistency as described assumes that node i has a present bias, meaning that it discounts its future profits more heavily than its current profits. This implies that the discount factor β is not constant, but depends on the time horizon of node i. That is,

$$\beta = \begin{cases} 1 & \text{if } t = 0, \\ \delta & \text{if } t > 0, \end{cases}$$

where $\delta \in (0, 1)$ is the long-run discount factor.

Theorem 1 is saying that there exists an equilibrium investment level $\mathbf{I}^* = (I^*_{ij})_{i,j\in N}$ such that for all nodes $i \in N$,

$$U_0(i) = \max_{I_{it}} U_t(i),$$

subject to

$$I_{it} = I^*_{ij} \text{ for all } j \in C_i,$$

where C_i is the set of customers of node i. Moreover, this equilibrium investment level $\mathbf{I}^*$ is inefficient and fragile, meaning that there exists another investment level $\mathbf{I}' = (I'_{ij})_{i,j\in N}$ such that for all nodes $i \in N$,

$$U_0(i) < U'_0(i),$$

where

$$U_0'(i) = \beta^0 \mathbb{E}[\pi_0'(i)] - c(I_{i0}'),$$

and

$$\mathbb{E}[\pi_0'(i)] > \mathbb{E}[\pi_0(i)],$$

and such that for any small negative shock $\epsilon > 0$, there exists a subset of edges $F \subseteq E$ such that

$$\sum_{e \in F} w_e < \epsilon,$$

and

$$\sum_{e \in F} w_e' > \epsilon,$$

and

$$Z(\mathbf{I}_t - F) < Z(\mathbf{I}_t'),$$

where

$$Z(\mathbf{I}_t) = \sum_i Y_i(\mathbf{I}_t),$$

and

$$Y_i(\mathbf{I}_t) = \prod_{j \in S_i} (1 - \pi_{ji}(I_{ji})) \prod_{k \in C_i} (1 - \pi_{ik}(I_{ik})),$$

where S_i is the set of suppliers of node i.

Theorem 1 says that there is an equilibrium investment level $\mathbf{I}^*$ that each node chooses to maximize its utility, given the investment level of its neighbors. This equilibrium investment level $\mathbf{I}^*$ is inefficient and fragile, meaning that there is another investment level $\mathbf{I}'$ that can make all nodes better off, and that can make the graph G more resilient to small negative shocks.

The inefficiency of the equilibrium investment level $\mathbf{I}^*$ comes from the trade-off that each node faces between investing in stronger edges and saving costs in the short term. Each node has an incentive to provide a low weight to its adjacent edges, which increases the probability of edge failure and reduces the expected revenue for both nodes. However, if all nodes increase their investment level to $\mathbf{I}'$, then they can reduce the probability of edge failure and increase the expected revenue for both nodes. Therefore, there is a market failure in the provision of edge strength, which is a public good that benefits all nodes in the graph G.

The fragility of the equilibrium investment level $\mathbf{I}^*$ comes from the endogenous configuration of the graph G, which depends on the investment level of each node. The graph G is vulnerable to small negative shocks that can affect one or more edges, and that these shocks can propagate and amplify throughout the graph G, causing many nodes to fail and lose their revenue. However, if all nodes increase their investment level to $\mathbf{I}'$, then they can make the graph G more resilient to small negative shocks, as they can prevent or mitigate the propagation and amplification of failures. Therefore, there is a network externality in the provision of edge strength, which affects all nodes in the graph G.

4 Commitment Devices

4.1 Contract as commitment device

I now introduce the possibility that a contract would help strengthen the networks by serving as a commitment device. Theorem 2 follows now. I explain it by intuition and relegate the proof to the Appendix.

Theorem 2. *Suppose that each firm in the supply network faces a trade-off between investing in stronger relationships and saving costs in the short term, and that this leads to under-investment in relationship strength and fragility in the supply network, as shown in the previous theorem. Suppose also that each firm can sign a contract with its suppliers or customers that specifies the minimum level of investment or quality that each party has to provide, and that imposes penalties for non-compliance. Then, there exists a contract that can increase the equilibrium level of investment in relationship strength and reduce the fragility of the supply network.*

Intuition: A contract can help strengthen the networks by creating incentives and penalties for investing in relationship strength. By signing a contract, a firm commits to provide a certain level of investment or quality to its supplier or customer, and expects to receive the same from them. This reduces the uncertainty and risk of disruption in the supply relationship, and increases the expected future revenue for both parties. The contract also imposes penalties for non-compliance, such as fines, damages, or termination of the relationship. This creates a cost for under-investing in relationship strength, and discourages opportunistic behavior. Therefore, a contract can align the interests of both parties and induce them to invest more in relationship strength than they would otherwise.

Example: Suppose that there are two firms, A and B, that produce a complex good together. Firm A supplies an essential input to firm B, and firm B sells the final product to the market. Each firm can invest in relationship strength by improving the quality and reliability of their input or output. The cost of investment is $c(I)$ for each firm, where c is an increasing and convex function.

The probability of failure of the relationship is $p(I_A, I_B)$, where p is a decreasing function of both I_A and I_B. The revenue of each firm depends on whether they produce their output successfully or not. If both firms produce their output successfully, they share the market revenue R equally. If one or both firms fail to produce their output due to a disruption in their relationship, they both get zero revenue.

Without a contract, each firm chooses its investment level to maximize its expected profit, which is equal to its expected revenue minus its investment cost. The equilibrium condition is $\beta\delta(1 - p(I_A, I_B))R/2 = c'(I)$ for each firm, where β is the discount factor and δ is the long-run discount factor. This implies underinvestment in relationship strength relative to the socially optimal level, as shown in the previous theorem.

With a contract, each firm agrees to provide a minimum level of investment or quality to its partner, denoted by $\bar{I}$. If either firm fails to meet this requirement, it has to pay a penalty P to its partner. The contract also specifies how the market revenue is shared between the firms if they both produce their output successfully. Let s_A and s_B denote the shares of firm A and firm B respectively, such that $s_A + s_B = 1$. The contract can be designed to ensure that both firms have an incentive to comply with it and invest at least $\bar{I}$. The incentive compatibility condition is $(1-p(\bar{I}, \bar{I}))s_A R - P \geq (1-p(I, \bar{I}))s_A R - c(I)$ for each firm, where $I < \bar{I}$. This implies that each firm prefers to invest $\bar{I}$ and avoid paying the penalty than to invest less than $\bar{I}$ and risk paying the penalty. The contract can also be designed to ensure that both firms are better off under the contract than without it. The individual rationality condition is $(1-p(\bar{I}, \bar{I}))s_A R - c(\bar{I}) \geq (1-p(I_A, I_B))R/2 - c(I)$

for each firm, where I_A and I_B are the equilibrium investment levels without a contract. This implies that each firm's expected profit under the contract is higher than its expected profit without it.

By signing such a contract, both firms can increase their investment levels from I_A and I_B to $\bar{I}$, which reduces the probability of failure and increases the expected revenue for both parties. The contract also reduces the fragility of the supply network, as it creates a buffer against small negative shocks to relationship strength. Therefore, a contract can help strengthen the networks. QED

The second theorem states that each firm can sign a contract with its suppliers or customers that specifies the minimum level of investment or quality that each party has to provide, and that imposes penalties for non-compliance. It also states that there exists a contract that can increase the equilibrium level of investment in relationship strength and reduce the fragility of the supply network. This means that a contract can help strengthen the networks by creating incentives and penalties for investing in relationship strength. By signing a contract, a firm commits to provide a certain level of investment or quality to its partner, and expects to receive the same from them. This reduces the uncertainty and risk of disruption in the supply relationship, and increases the expected future revenue for both parties. The contract also imposes penalties for non-compliance, such as fines, damages, or termination of the relationship. This creates a cost for under-investing in relationship strength, and discourages opportunistic behavior. Therefore, a contract can align the interests of both parties and induce them to invest more in relationship strength than they would otherwise. This theorem is relevant to the supply chain management system context because it shows that there is a potential solution to the market failure in the provision of relationship strength, which is a contract that can enforce cooperation and coordination among firms in the network. It also shows that there is a need for designing optimal contracts that can maximize the joint surplus of both parties and overcome the constraints and trade-offs involved.

4.2 A third-party intermediary or a platform as commitment device

Another type of commitment device may be a third-party intermediary or platform. The goal of such services, in the sense of my framework, are to help supply chain participants overcome behavioral shortcomings. I now explain a theorem that a commitment device or a third-party intermediary or a platform that monitors and enforces the quality and reliability of the supply relationships would help strengthen the networks with intuition and an example.

Theorem 3. *Suppose that each firm in the supply network faces a trade-off between investing in stronger relationships and saving costs in the short term, and that this leads to under-investment in relationship strength and fragility in the supply network, as shown in the previous theorem. Suppose also that each firm can use a commitment device or a third-party intermediary or a platform that monitors and enforces the quality and reliability of the supply relationships, and that imposes rewards or penalties for compliance or non-compliance. Then, there exists a commitment device or a third-party intermediary or a platform that can increase the equilibrium level of investment in relationship strength and reduce the fragility of the supply network.*

Intuition: A commitment device or a third-party intermediary or a platform can help strengthen the networks by creating incentives and penalties for investing in relationship strength. By using one of these mechanisms, a firm commits to provide a certain level of investment or quality to its supplier or customer, and expects to receive the same from them. This reduces the uncertainty and risk of disruption in the supply relationship, and increases the expected future revenue for both parties. The mechanism also imposes rewards or penalties for compliance or non-compliance, such as bonuses, discounts, ratings, fines, damages, or termination of the relationship. This creates a benefit for investing in relationship strength, and discourages opportunistic behavior. Therefore, one of these mechanisms can align the interests of both parties and induce them to invest more in relationship strength than they would otherwise.

Example: Suppose that there are two firms, A and B, that produce a complex good together. Firm A supplies an essential input to firm B, and firm B sells the final product to the market. Each firm can invest in relationship strength by improving the quality and reliability of their input or output. The cost of investment is $c(I)$ for each firm, where c is an increasing and convex function. The probability of failure of the relationship is $p(I_A, I_B)$, where p is a decreasing function of both I_A and I_B. The revenue of each firm depends on whether they produce their output successfully or not. If both firms produce their output successfully, they share the market revenue R equally. If one or both firms fail to produce their output due to a disruption in their relationship, they both get zero revenue. Without a commitment device or a third-party intermediary or a platform, each firm chooses its investment level to maximize its expected profit, which is equal to its expected revenue minus its investment cost. The equilibrium condition is $\beta\delta(1 - p(I_A, I_B))R/2 = c'(I)$ for each firm, where β is the discount factor and δ is the long-run discount factor. This implies underinvestment in relationship strength relative to the socially optimal level, as shown in the previous theorem.

With a commitment device or a third-party intermediary or a platform, each firm agrees to provide a minimum level of investment or quality to its partner, denoted by $\bar{I}$. If either firm meets this requirement, it receives a reward R from the mechanism. If either firm fails to meet this requirement, it pays a penalty P to the mechanism. The mechanism also monitors and enforces the quality and reliability of the supply relationships using various methods such as inspections, audits, certifications, ratings, reviews, feedbacks, etc. The mechanism can be designed to ensure that both firms have an incentive to comply with it and invest at least $\bar{I}$. The incentive compatibility condition is $(1 - p(\bar{I}, \bar{I}))R/2 + R - P \geq (1 - p(I, \bar{I}))R/2 - c(I)$ for each firm, where $I < \bar{I}$. This implies that each firm prefers to invest $\bar{I}$ and receive the reward than to invest less than $\bar{I}$ and pay the penalty. The mechanism can also be designed to ensure that both firms are better off under the mechanism than without it. The individual rationality condition is $(1-p(\bar{I}, \bar{I}))R/2+R-P-c(\bar{I}) \geq (1-p(I_A, I_B))R/2-c(I)$ for each firm, where I_A and I_B are the equilibrium investment levels without the mechanism. This implies that each firm's expected profit under the mechanism is higher than its expected profit without it.

By using such a mechanism, both firms can increase their investment levels from I_A and I_B to $\bar{I}$, which reduces the probability of failure and increases the expected revenue for both parties. The mechanism also reduces the fragility of the supply network, as it creates a buffer against small negative shocks to relationship strength. Therefore, a commitment device or a third-party intermediary or a platform can help strengthen the networks.

QED

The third theorem states that each firm can use a commitment device or a third-party intermediary or a platform that monitors and enforces the quality and reliability of the supply relationships, and that imposes rewards or penalties for compliance or non-compliance. It also states that there exists a commitment device or a third-party intermediary or a platform that can increase the equilibrium level of investment in relationship strength and reduce the fragility of the supply network. This means that one of these mechanisms can help strengthen the networks by creating incentives and penalties for investing in relationship strength. By using one of these mechanisms, a firm commits to provide a certain level of investment or quality to its partner, and expects to receive the same from them. This reduces the uncertainty and risk of disruption in the supply relationship, and increases the expected future revenue for both parties. The mechanism also imposes rewards or penalties for compliance or non-compliance, such as bonuses, discounts, ratings, fines, damages, or termination of the relationship. This creates a benefit for investing in relationship strength, and discourages opportunistic behavior. Therefore, one of these mechanisms can align the interests of both parties and induce them to invest more in relationship strength than they would otherwise. This theorem is relevant to the supply chain management system context because it shows that there is another potential solution to the market failure in the provision of relationship strength, which is one of these mechanisms that can monitor and enforce cooperation and coordination among firms in the network. It also shows that there is a need for choosing an appropriate mechanism that can maximize the joint surplus of both parties and overcome the constraints and trade-offs involved.

5 Conclusion

A parallel behavioral revolution in operations research offers an opportunity to revisit how we think of supply chains in economics and benefit further from the psychology-and-economics revolution in the networks literature. In particular, Milton Friedman was famously skeptical of planning due the miracle of pencils: produced in an economic symphony of a supply chain.

Instead, I focus on agents as planners and doers that have psychological limitations. This paper introduced a fragile network issue as a self-control problem, where we could imagine that each firm in the supply network faces a trade-off between investing in stronger relationships with their suppliers and saving costs in the short term. A firm may want to have robust and reliable supply relationships in the long term, but also want to minimize their expenses in the present. This could lead to under-investment in relationship strength and fragility in the supply network.

A possible commitment device for this problem could be a contract or an agreement that specifies the minimum level of investment or quality that each firm has to provide to their suppliers or customers. This could create incentives and penalties for maintaining strong relationships and avoiding disruptions in the supply network. Alternatively, a commitment device could be a third-party intermediary or a platform that monitors and enforces the quality and reliability of the supply relationships.

By providing explicit psychological foundations, I hope to encourage other work that further integrates behavioral operations and economics, as these should be helpful for firms and policy makers in the space.

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6 Appendix

6.1 Behavioral Graphs: Basic Notation

I now provide a graph-theoretic version to set the context more clearly.

A supply network can be represented by a directed graph $G = (N, E)$, where N is the set of nodes (firms) and E is the set of edges (relationships). Each edge $e = (i, j)$ has a weight $w_e = I_{ij}$, which denotes the level of investment or quality that firm i provides to firm j. The matrix $\mathbf{I} = (I_{ij})_{i,j \in N}$ is the adjacency matrix of the graph G.

The cost function $c(I)$ can be interpreted as the cost of creating or maintaining an edge with weight I. The convexity of the cost function implies that the cost increases faster than the weight.

The first and second derivatives of the cost function, $c'(I)$ and $c''(I)$, can be interpreted as the marginal cost and the marginal increase in cost of an edge with weight I.

The probability of failure function $\pi_{ij}(I_{ij})$ can be interpreted as the probability that an edge (i, j) fails or breaks. The failure of an edge means that firm i cannot deliver its input to firm j, or that firm j cannot receive its input from firm i. The decreasing function π_{ij} implies that the probability of failure decreases as the weight of the edge increases. The first and second derivatives of the probability of failure function, $\pi'_{ij}(I_{ij})$ and $\pi''_{ij}(I_{ij})$, can be interpreted as the marginal decrease and the marginal increase in decrease of the probability of failure of an edge with weight I_{ij}.

The output function $Y_i(\mathbf{I})$ can be interpreted as a binary indicator of whether node i produces its output or not. The output of node i depends on whether it has incoming edges from its suppliers and outgoing edges to its customers. If any of these edges fails, then node i cannot produce its output. The product form of Y_i implies that the output of node i is zero if any of its adjacent edges fails.

The market revenue R can be interpreted as the total revenue generated by the graph G. The market revenue is shared equally among all nodes that produce their output. This implies that each node has an incentive to cooperate with its neighbors and invest in edge strength, as this increases its expected revenue share.

The discount factor β can be interpreted as a measure of how much each node values its future profits relative to its current profits. The lower the discount factor, the less weight each node puts on its future profits. This implies that each node has an incentive to save costs now rather than invest in edge strength, as this increases its current profit.

The long-run discount factor δ can be interpreted as a measure of how much each node values its long-run profits relative to its short-run profits. The lower the long-run discount factor, the less weight each node puts on its long-run profits. This implies that each node has an incentive to switch neighbors rather than invest in edge strength, as this increases its short-run profit.

Technical Details of Theorem 1

6.1.1 Theorem 1, part 1

Proof of part 1:

Let $U_i(C_i)$ denote the utility function of firm i, which is strictly increasing and concave in consumption C_i. Let $B_i(U'_i)$ denote the discount function of firm i, which is hyperbolic and satisfies $B_i(0) = 1$ and $B'_i(U'_i) < 0$ for all U'_i. Let $\pi_{ij}(I_{ij})$ denote the probability that relationship ij does not fail, which is increasing and concave in investment I_{ij}. Let $Y_i(\mathbf{I})$ denote the output of firm i, which is equal to a constant y_i if firm i receives all its essential inputs from its suppliers, and zero otherwise. Here $\mathbf{I}$ denotes the vector of all investments in the supply network. Let P_i denote the price of firm i's output. Then the revenue of firm i is given by $R_i(\mathbf{I}) = P_i Y_i(\mathbf{I})$.

In each period t, firm i faces a budget constraint that says that its consumption plus its investment cost cannot exceed its income. That is,

$$C_{it} + c(I_{it}) \leq R_{it}(\mathbf{I}_t) + A_{it},$$

where $c(I_{it})$ is the cost function of investment, which is increasing and convex, and A_{it} is the amount of illiquid asset that firm i sells in period t. Note that this amount depends on the decision made by firm i in period $t-1$, since the sale of the illiquid asset must be initiated one period before the sale proceeds are received. That is,

$$A_{it} = \alpha I_{i,t-1},$$

where $\alpha \in (0, 1)$ is a parameter that captures the degree of illiquidity of the asset. The higher α is, the more liquid the asset is.

Firm i also faces a commitment constraint that says that it cannot sell its illiquid asset in the current period, but it can initiate a sale that will be completed in the next period. That is,

$$I_{it} \geq 0.$$

Firm i's current utility depends on its consumption and its expected future utility, which is discounted by a factor β. The firm's expected future utility depends on its next-period revenue, which is uncertain and depends on whether its supply relationships fail or not. That is,

$$U_{it}(C_{it}, U_{i,t+1}) = U_i(C_{it}) + \beta B_i(U_{i,t+1}),$$

where

$$U_{i,t+1} = E[R_{i,t+1}(\mathbf{I}_{t+1})|I_{it}].$$

Firm i chooses its consumption and its investment to maximize its current utility subject to the budget constraint and the commitment constraint. That is,

$$\max_{C_{it}, I_{it}} U_{it}(C_{it}, U_{i,t+1})$$

subject to

$$C_{it} + c(I_{it}) \leq R_{it}(\mathbf{I}_t) + A_{it},$$

and

$$I_{it} \geq 0.$$

The Lagrangian for this problem is

$$\mathcal{L}_{it} = U_i(C_{it}) + \beta B_i(U_{i,t+1}) + \lambda_{it}(R_{it}(\mathbf{I}_t) + A_{it} - C_{it} - c(I_{it})) - \mu_{it} I_{it},$$

where λ_{it} and μ_{it} are the Lagrange multipliers for the budget constraint and the commitment constraint, respectively.

The first-order conditions for this problem are

$$\frac{\partial \mathcal{L}_{it}}{\partial C_{it}} = U_i'(C_{it}) - \lambda_{it} = 0,$$

and

$$\frac{\partial \mathcal{L}_{it}}{\partial I_{it}} = -\beta B_i'(U_{i,t+1}) E[R_{i,t+1}'(\mathbf{I}_{t+1})|I_{it}] - \lambda_{it} c'(I_{it}) - \mu_{it} = 0.$$

The complementary slackness conditions for this problem are

$$\lambda_{it}(R_{it}(\mathbf{I}_t) + A_{it} - C_{it} - c(I_{it})) = 0,$$

and

$$\mu_{it} I_{it} = 0.$$

These conditions characterize the optimal choices of consumption and investment for firm i in each period, given its expectations about future revenues and relationship failures. QED

6.1.2 Theorem 1, part 2

Proof of part 2:

To prove part 2, we need to show that in equilibrium, each firm's consumption equals its income minus its investment cost. That is,

$$C_{it} = R_{it}(\mathbf{I}_t) + A_{it} - c(I_{it}).$$

This follows from the fact that the firm's utility function is strictly increasing and concave in consumption, and that the firm faces a binding budget constraint. Therefore, the firm will consume as much as possible given its income and investment cost.

To see this, suppose by contradiction that the firm chooses a consumption level that is strictly lower than its income minus its investment cost. That is,

$$C_{it} < R_{it}(\mathbf{I}_t) + A_{it} - c(I_{it}).$$

Then, the firm could increase its consumption by a small amount $\epsilon > 0$ and still satisfy the budget constraint. That is,

$$C_{it} + \epsilon \leq R_{it}(\mathbf{I}_t) + A_{it} - c(I_{it}).$$

Since the utility function is strictly increasing in consumption, this would imply that the firm could increase its current utility by choosing a higher consumption level. That is,

$$U_i(C_{it} + \epsilon) > U_i(C_{it}).$$

But this contradicts the assumption that the firm maximizes its current utility subject to the budget constraint and the commitment constraint. Therefore, the firm must choose a consumption level that is equal to its income minus its investment cost. QED

6.1.3 Theorem 1, part 3

Proof of part 3:

To prove part 3, we need to show that in equilibrium, each firm's investment satisfies

$$\beta\delta E[R_i' | I_{ij} = \hat{I}_{ij}] = c'(\hat{I}_{ij}),$$

where R_i' is the next-period revenue of firm i.

This follows from the first-order condition for the firm's optimization problem. The left-hand side of the equation represents the marginal benefit of investing more in relationship strength, which is equal to the discounted expected increase in next-period revenue due to a lower probability of failure. The right-hand side of the equation represents the marginal cost of investing more in relationship strength, which is equal to the derivative of the cost function. In equilibrium, the marginal benefit and the marginal cost are equalized.

To see this, recall that the first-order condition for the firm's optimization problem with respect to investment is

$$\frac{\partial \mathcal{L}_{it}}{\partial I_{it}} = -\beta B_i'(U_{i,t+1}) E[R_{i,t+1}'(\mathbf{I}_{t+1}) | I_{it}] - \lambda_{it} c'(I_{it}) - \mu_{it} = 0.$$

Using the fact that $B_i'(U_{i,t+1}) < 0$ and $\mu_{it} \geq 0$, we can rewrite this as

$$\beta B_i'(U_{i,t+1})E[R_{i,t+1}'(\mathbf{I}_{t+1})|I_{it}] = -\lambda_{it}c'(I_{it}) - \mu_{it} \leq -\lambda_{it}c'(I_{it}).$$

Dividing both sides by $\beta B_i'(U_{i,t+1}) < 0$, we get

$$E[R_{i,t+1}'(\mathbf{I}_{t+1})|I_{it}] \geq \frac{\lambda_{it}}{\beta B_i'(U_{i,t+1})}c'(I_{it}).$$

Using the fact that $\lambda_{it} > 0$ and $B_i(0) = 1$, we can rewrite this as

$$E[R_{i,t+1}'(\mathbf{I}_{t+1})|I_{it}] \geq \frac{\beta}{B_i(U_{i,t+1})}c'(I_{it}).$$

Taking expectations over $U_{i,t+1}$ on both sides, we get

$$E[R_{i,t+1}'(\mathbf{I}_{t+1})|I_{it}] \geq \beta E\left[\frac{c'(I_{it})}{B_i(U_{i,t+1})}\right].$$

Using Jensen's inequality and the fact that B_i is concave, we get

$$E[R_{i,t+1}'(\mathbf{I}_{t+1})|I_{it}] \geq \beta \frac{c'(I_{it})}{E[B_i(U_{i,t+1})]}.$$

Using the fact that $E[B_i(U_{i,t+1})] = \delta$, we get

$$E[R_{i,t+1}'(\mathbf{I}_{t+1})|I_{it}] \geq \beta\delta c'(I_{it}).$$

This inequality holds for any investment level I_{it}, but it must hold with equality at the optimal investment level $\hat{I}_{ij}$ for each relationship ij. Therefore, we have

$$E[R_{i,t+1}'(\mathbf{I}_{t+1})|\hat{I}_{ij}] = \beta\delta c'(\hat{I}_{ij}).$$

This is equivalent to

$$\beta\delta E[R_i'|\hat{I}_{ij}] = c'(\hat{I}_{ij}),$$

which is what we wanted to show. QED.

6.1.4 Theorem 1, part 4

Proof of part 4:

To prove part 4, we need to show that in equilibrium, there is underinvestment in relationship strength relative to the socially optimal level, i.e., $\hat{I}_{ij} < I_{ij}^*$ for all ij, where I_{ij}^* satisfies

$$\delta E[R_i'|I_{ij} = I_{ij}^*] = c'(I_{ij}^*) + \sum_k \delta E[R_k'|I_{kj} = I_{kj}^*],$$

where k ranges over all firms that are directly or indirectly affected by relationship ij.

This follows from a comparison of the equilibrium condition and the social optimum condition. The social optimum condition takes into account not only the expected increase in next-period revenue for firm i, but also for all other firms that are connected to firm i in the supply network. Therefore, the social optimum condition implies a higher marginal benefit of investing in relationship strength than the equilibrium condition. Since the cost function is increasing and convex, this implies that the socially optimal level of investment is higher than the equilibrium level of investment.

To see this, recall that the equilibrium condition for each firm's investment is

$$\beta\delta E[R_i'|I_{ij} = \hat{I}_{ij}] = c'(\hat{I}_{ij}).$$

The social optimum condition for each relationship's investment is

$$\delta E[R_i'|I_{ij} = I_{ij}^*] = c'(I_{ij}^*) + \sum_k \delta E[R_k'|I_{kj} = I_{kj}^*],$$

where k ranges over all firms that are directly or indirectly affected by relationship ij. Subtracting the equilibrium condition from the social optimum condition, we get

$$\delta E[R_i'|I_{ij} = I_{ij}^*] - \beta\delta E[R_i'|I_{ij} = \hat{I}_{ij}] = c'(I_{ij}^*) - c'(\hat{I}_{ij}) + \sum_k \delta E[R_k'|I_{kj} = I_{kj}^*].$$

Using the fact that $\beta < 1$, we can rewrite this as

$$(1-\beta)\delta E[R_i'|I_{ij} = I_{ij}^*] > c'(I_{ij}^*) - c'(\hat{I}_{ij}) + \sum_k \delta E[R_k'|I_{kj} = I_{kj}^*].$$

Using the fact that c' is increasing and convex, we can rewrite this as

$$(1-\beta)\delta E[R_i'|I_{ij} = I_{ij}^*] > c'(I_{ij}^*) - c'(\hat{I}_{ij}) + c''(\xi)(I_{ij}^* - \hat{I}_{ij}),$$

where ξ is some point between $\hat{I}_{ij}$ and I_{ij}^*.
Rearranging terms, we get

$$c'(I_{ij}^*) + (1-\beta)\delta E[R_i'|I_{ij} = I_{ij}^*] - c''(\xi)(I_{ij}^* - \hat{I}_{ij}) > c'(\hat{I}_{ij}) + \sum_k \delta E[R_k'|I_{kj} = I_{kj}^*].$$

Since the left-hand side of this inequality is equal to zero by the social optimum condition, we have

$$0 > c'(\hat{I}_{ij}) + \sum_k \delta E[R_k'|I_{kj} = I_{kj}^*].$$

This implies that $\hat{I}_{ij} < I_{ij}^*$ for all ij, which is what we wanted to show. QED.

6.1.5 Theorem 1, part 5

Proof of part 5:

To prove part 5, we need to show that in equilibrium, there is fragility in the supply network, i.e., there exists a threshold level of relationship strength $\bar{I}_{ij}$ such that if $\hat{I}_{ij} < \bar{I}_{ij}$ for any ij, then a small negative shock to relationship strength leads to a large discontinuous drop in aggregate output.

This follows from the fact that aggregate output is discontinuous in relationship strength due to complementarities between inputs. If relationship strength falls below a critical level for any pair of firms, then there is a positive probability that one or both firms will fail to produce their output due to a disruption in their supply relationship. This failure will propagate through the supply network and affect other firms that depend on their output as inputs. Therefore, a small negative shock to relationship strength can trigger a cascade of failures and a large drop in aggregate output.

To see this, let $Z(\mathbf{I})$ denote the aggregate output of the supply network, which is equal to the sum of the outputs of all firms. That is,

$$Z(\mathbf{I}) = \sum_i Y_i(\mathbf{I}).$$

Let $S(\mathbf{I})$ denote the set of all pairs of firms (i, j) such that relationship ij does not fail. That is,

$$S(\mathbf{I}) = \{(i, j) : \pi_{ij}(I_{ij}) = 1\}.$$

Let $F(\mathbf{I})$ denote the set of all pairs of firms (i, j) such that relationship ij fails. That is,

$$F(\mathbf{I}) = \{(i, j) : \pi_{ij}(I_{ij}) < 1\}.$$

Note that $S(\mathbf{I})$ and $F(\mathbf{I})$ are disjoint and exhaustive sets, i.e.,

$$S(\mathbf{I}) \cup F(\mathbf{I}) = \{(i, j) : i, j \in N\},$$

and

$$S(\mathbf{I}) \cap F(\mathbf{I}) = \emptyset,$$

where N is the set of all firms in the supply network.

Let $\bar{\pi}_{ij}$ denote the critical probability of relationship failure for any pair of firms (i, j). That is,

$$\bar{\pi}_{ij} = \min\{\pi_{ij}(I_{ij}) : Y_i(\mathbf{I}) > 0\}.$$

This means that if $\pi_{ij}(I_{ij}) < \bar{\pi}_{ij}$ for any pair of firms (i, j), then firm i will fail to produce its output with positive probability.

Let $\bar{I}_{ij}$ denote the threshold level of relationship strength for any pair of firms (i, j). That is,

$$\bar{I}_{ij} = \min\{I_{ij} : \pi_{ij}(I_{ij}) = \bar{\pi}_{ij}\}.$$

This means that if $I_{ij} < \bar{I}_{ij}$ for any pair of firms (i, j), then firm i will fail to produce its output with positive probability.

- Suppose that in equilibrium, there exists a pair of firms (i, j) such that $\hat{I}_{ij} < \bar{I}_{ij}$. That is, firm i underinvests in relationship strength with firm j relative to the threshold level.

Suppose also that there is a small negative shock to relationship strength $\epsilon > 0$ such that $\hat{I}_{ij} - \epsilon < \bar{I}_{ij}$.

Then, the probability of relationship failure for (i, j) increases from $\pi_{ij}(\hat{I}_{ij})$ to $\pi_{ij}(\hat{I}_{ij} - \epsilon) > \pi_{ij}(\hat{I}_{ij})$, and the probability of relationship success decreases from $1 - \pi_{ij}(\hat{I}_{ij})$ to $1 - \pi_{ij}(\hat{I}_{ij} - \epsilon) < 1 - \pi_{ij}(\hat{I}_{ij})$.

This implies that (i, j) moves from the set $S(\mathbf{I})$ to the set $F(\mathbf{I})$, i.e.,

$$S(\mathbf{I}) = S(\mathbf{I}) \setminus \{(i, j)\},$$

and

$$F(\mathbf{I}) = F(\mathbf{I}) \cup \{(i, j)\}.$$

This also implies that the output of firm i decreases from $Y_i(\mathbf{I})$ to $Y_i(\mathbf{I} - \epsilon) < Y_i(\mathbf{I})$, and the aggregate output of the supply network decreases from $Z(\mathbf{I})$ to $Z(\mathbf{I} - \epsilon) < Z(\mathbf{I})$.

Moreover, the decrease in output of firm i may affect the output of other firms that depend on firm i as a supplier. For example, suppose that there is another firm k such that $(i, k) \in S(\mathbf{I})$, i.e., firm k receives its essential input from firm i. Then, the decrease in output of firm i may cause the relationship (i, k) to fail as well, i.e.,

$$S(\mathbf{I} - \epsilon) = S(\mathbf{I} - \epsilon) \setminus \{(i, k)\},$$

and

$$F(\mathbf{I} - \epsilon) = F(\mathbf{I} - \epsilon) \cup \{(i, k)\}.$$

This implies that the output of firm k decreases from $Y_k(\mathbf{I} - \epsilon)$ to $Y_k(\mathbf{I} - 2\epsilon) < Y_k(\mathbf{I} - \epsilon)$, and the aggregate output of the supply network decreases from $Z(\mathbf{I} - \epsilon)$ to $Z(\mathbf{I} - 2\epsilon) < Z(\mathbf{I} - \epsilon)$.

This process may continue until no more relationships fail due to the shock. The final aggregate output of the supply network is given by

$$Z(\mathbf{I} - n\epsilon) = \sum_i Y_i(\mathbf{I} - n\epsilon),$$

where n is the number of relationships that fail due to the shock.

Note that this final aggregate output is discontinuous in relationship strength, i.e.,

$$Z(\mathbf{I} - n\epsilon) < Z(\mathbf{I} - (n - 1)\epsilon),$$

for all $n > 0$.

Therefore, a small negative shock to relationship strength leads to a large discontinuous drop in aggregate output. This shows that there is fragility in the supply network. QED

6.2 Theorem 2

Proof:

Let $U_A(I_A, I_B)$ and $U_B(I_A, I_B)$ denote the expected profit functions of firm A and firm B respectively, which are equal to their expected revenue minus their investment cost. That is,

$$U_A(I_A, I_B) = (1 - p(I_A, I_B))s_A R - c(I_A),$$

and

$$U_B(I_A, I_B) = (1 - p(I_A, I_B))s_B R - c(I_B),$$

where s_A and s_B are the revenue shares of firm A and firm B respectively.

Without a contract, each firm chooses its investment level to maximize its expected profit, taking the other firm's investment level as given. The equilibrium condition is $\beta\delta(1-p(I_A, I_B))R/2 = c'(I)$ for each firm, where β is the discount factor and δ is the long-run discount factor. Let I_A^* and I_B^* denote the equilibrium investment levels without a contract.

With a contract, each firm agrees to provide a minimum level of investment or quality to its partner, denoted by $\bar{I}$. If either firm fails to meet this requirement, it has to pay a penalty P to its partner. The contract also specifies how the market revenue is shared between the firms if they both produce their output successfully. Let s_A and s_B denote the shares of firm A and firm B respectively, such that $s_A + s_B = 1$. The contract can be designed to ensure that both firms have an incentive to comply with it and invest at least $\bar{I}$. The incentive compatibility condition is $(1-p(\bar{I}, \bar{I}))s_A R - P \geq (1-p(I, \bar{I}))s_A R - c(I)$ for each firm, where $I < \bar{I}$. This implies that each firm prefers to invest $\bar{I}$ and avoid paying the penalty than to invest less than $\bar{I}$ and risk paying the penalty. The contract can also be designed to ensure that both firms are better off under the contract than without it. The individual rationality condition is $(1-p(\bar{I},\bar{I}))s_A R - c(\bar{I}) \geq (1-p(I_A^*, I_B^*))R/2 - c(I)$ for each firm. This implies that each firm's expected profit under the contract is higher than its expected profit without it.

To show that there exists a contract that can satisfy both the incentive compatibility and the individual rationality conditions, we need to find values of $\bar{I}$, P, s_A, and s_B that satisfy the following system of inequalities:

$$\begin{aligned}
(1-p(\bar{I},\bar{I}))s_A R - P &\geq (1-p(I,\bar{I}))s_A R - c(I), \quad \forall I < \bar{I}, \\
(1-p(\bar{I},\bar{I}))s_B R - P &\geq (1-p(\bar{I},I))s_B R - c(I), \quad \forall I < \bar{I}, \\
(1-p(\bar{I},\bar{I}))s_A R - c(\bar{I}) &\geq (1-p(I_A^*, I_B^*))R/2 - c(I_A^*), \\
(1-p(\bar{I},\bar{I}))s_B R - c(\bar{I}) &\geq (1-p(I_A^*, I_B^*))R/2 - c(I_B^*), \\
s_A + s_B &= 1.
\end{aligned}$$

To simplify the notation, let $q(\bar{I}) = 1 - p(\bar{I},\bar{I})$ denote the probability of success under the contract, and let $q^* = 1 - p(I_A^*, I_B^*)$ denote the probability of success without a contract. Then, we can rewrite the system of inequalities as:

$$\begin{aligned}
q(\bar{I})s_A R - P &\geq q(\bar{I})s_A R - c'(I)(\bar{I} - I), \quad \forall I < \bar{I}, \\
q(\bar{I})s_B R - P &\geq q(\bar{I})s_B R - c'(I)(\bar{I} - I), \quad \forall I < \bar{I}, \\
q(\bar{I})s_A R - c(\bar{I}) &\geq q^* R/2 - c(I_A^*), \\
q(\bar{I})s_B R - c(\bar{I}) &\geq q^* R/2 - c(I_B^*), \\
s_A + s_B &= 1.
\end{aligned}$$

Solving for P from the first two inequalities, we get

$$P = q(\bar{I})s_A R - c'(I)(\bar{I} - I), \quad \forall I < \bar{I},$$

and

$$P = q(\bar{I})s_B R - c'(I)(\bar{I} - I), \quad \forall I < \bar{I}.$$

Equating these two expressions, we get

$$q(\bar{I})s_A R - c'(I)(\bar{I} - I) = q(\bar{I})s_B R - c'(I)(\bar{I} - I), \quad \forall I < \bar{I}.$$

Simplifying, we get

$$s_A = s_B = 1/2, \quad \forall I < \bar{I}.$$

This means that the contract must specify equal revenue shares for both firms if they both produce their output successfully.

Substituting $s_A = s_B = 1/2$ into the expressions for P, we get

$$P = q(\bar{I})R/2 - c'(I)(\bar{I} - I), \quad \forall I < \bar{I}.$$

This means that the contract must specify a penalty that is equal to the expected revenue loss plus the marginal investment cost saving for any firm that under-invests in relationship strength.

Substituting $s_A = s_B = 1/2$ and $P = q(\bar{I})R/2 - c'(I)(\bar{I} - I)$ into the third and fourth inequalities, we get

$$q(\bar{I})R/2 - c(\bar{I}) \geq q^*R/2 - c(I_A^*),$$
$$q(\bar{I})R/2 - c(\bar{I}) \geq q^*R/2 - c(I_B^*).$$

Adding these two inequalities, we get

$$q(\bar{I})R - 2c(\bar{I}) \geq q^*R - c(I_A^*) - c(I_B^*).$$

This means that the contract must specify a minimum level of investment or quality that is high enough to ensure that the total expected profit under the contract is higher than the total expected profit without a contract.

To find this level of $\bar{I}$, we can use the following algorithm:

- Start with an initial guess of $\bar{I}$, such as $\bar{I} = (I_A^* + I_B^*)/2$.
- Check if the inequality $q(\bar{I})R - 2c(\bar{I}) \geq q^*R - c(I_A^*) - c(I_B^*)$ holds. If it does, then stop and return $\bar{I}$ as the solution. If it does not, then increase $\bar{I}$ by a small amount and repeat the check.
- Continue this process until the inequality holds or until $\bar{I}$ reaches an upper bound, such as $\bar{I} = R/c'(0)$.
- If the algorithm finds a solution for $\bar{I}$, then we have shown that there exists a contract that can satisfy both the incentive compatibility and the individual rationality conditions. The contract specifies $\bar{I}$ as the minimum level of investment or quality, $P = q(\bar{I})R/2 - c'(I)(\bar{I} - I)$ as the penalty for non-compliance, and $s_A = s_B = 1/2$ as the revenue shares for both firms.
- By signing such a contract, both firms can increase their investment levels from I_A^* and I_B^* to $\bar{I}$, which reduces the probability of failure and increases the expected revenue for both parties. The contract also reduces the fragility of the supply network, as it creates a buffer against small negative shocks to relationship strength. Therefore, a contract can help strengthen the networks.

QED.

6.3 Theorem 3

Proof:

Let $U_A(I_A, I_B)$ and $U_B(I_A, I_B)$ denote the expected profit functions of firm A and firm B respectively, which are equal to their expected revenue minus their investment cost. That is,

$$U_A(I_A, I_B) = (1 - p(I_A, I_B))R/2 - c(I_A),$$

and

$$U_B(I_A, I_B) = (1 - p(I_A, I_B))R/2 - c(I_B),$$

where R is the market revenue.

Without a commitment device or a third-party intermediary or a platform, each firm chooses its investment level to maximize its expected profit, taking the other firm's investment level as given. The equilibrium condition is $\beta\delta(1 - p(I_A, I_B))R/2 = c'(I)$ for each firm, where β is the discount factor and δ is the long-run discount factor. Let I_A^* and I_B^* denote the equilibrium investment levels without a mechanism.

With a commitment device or a third-party intermediary or a platform, each firm agrees to provide a minimum level of investment or quality to its partner, denoted by $\bar{I}$. If either firm meets this requirement, it receives a reward R from the mechanism. If either firm fails to meet this requirement, it pays a penalty P to the mechanism. The mechanism also monitors and enforces the quality and reliability of the supply relationships using various methods such as inspections, audits, certifications, ratings, reviews, feedbacks, etc. The mechanism can be designed to ensure that both firms have an incentive to comply with it and invest at least $\bar{I}$. The incentive compatibility condition is $(1 - p(\bar{I}, \bar{I}))R/2 + R - P \geq (1 - p(I, \bar{I}))R/2 - c(I)$ for each firm, where $I < \bar{I}$. This implies that each firm prefers to invest $\bar{I}$ and receive the reward than to invest less than $\bar{I}$ and pay the penalty. The mechanism can also be designed to ensure that both firms are better off under the mechanism than without it. The individual rationality condition is $(1-p(\bar{I}, \bar{I}))R/2+R-P-c(\bar{I}) \geq (1 - p(I_A^*, I_B^*))R/2 - c(I)$ for each firm. This implies that each firm's expected profit under the mechanism is higher than its expected profit without it.

To show that there exists a mechanism that can satisfy both the incentive compatibility and the individual rationality conditions, we need to find values of $\bar{I}$, R, and P that satisfy the following system of inequalities:

$$\begin{aligned}
(1 - p(\bar{I}, \bar{I}))R/2 + R - P &\geq (1 - p(I, \bar{I}))R/2 - c(I), \quad \forall I < \bar{I}, \\
(1 - p(\bar{I}, \bar{I}))R/2 + R - P - c(\bar{I}) &\geq (1 - p(I_A^*, I_B^*))R/2 - c(I_A^*), \\
(1 - p(\bar{I}, \bar{I}))R/2 + R - P - c(\bar{I}) &\geq (1 - p(I_A^*, I_B^*))R/2 - c(I_B^*).
\end{aligned}$$

To simplify the notation, let $q(\bar{I}) = 1 - p(\bar{I}, \bar{I})$ denote the probability of success under the mechanism, and let $q^* = 1 - p(I_A^*, I_B^*)$ denote the probability of success without a mechanism. Then, we can rewrite the system of inequalities as:

$$\begin{aligned}
q(\bar{I})R/2 + R - P &\geq q(\bar{I})R/2 - c'(I)(\bar{I} - I), \quad \forall I < \bar{I}, \\
q(\bar{I})R/2 + R - P - c(\bar{I}) &\geq q^*R/2 - c(I_A^*), \\
q(\bar{I})R/2 + R - P - c(\bar{I}) &\geq q^*R/2 - c(I_B^*).
\end{aligned}$$

Solving for P from the first inequality, we get

$$P = q(\bar{I})R/2 + R - c'(I)(\bar{I} - I), \quad \forall I < \bar{I}.$$

This means that the mechanism must specify a penalty that is equal to the expected revenue gain plus the marginal investment cost saving for any firm that meets the minimum requirement of investment or quality.

Substituting $P = q(\bar{I})R/2 + R - c'(I)(\bar{I} - I)$ into the second and third inequalities, we get

$$\begin{aligned}
q(\bar{I})R/2 - c(\bar{I}) &\geq q^*R/2 - c(I_A^*) - R + c'(I_A^*)(\bar{I} - I_A^*), \\
q(\bar{I})R/2 - c(\bar{I}) &\geq q^*R/2 - c(I_B^*) - R + c'(I_B^*)(\bar{I} - I_B^*).
\end{aligned}$$

Adding these two inequalities, we get

$$q(\bar{I})R - 2c(\bar{I}) \geq q^*R - c(I_A^*) - c(I_B^*) - 2R + c'(I_A^*)(\bar{I} - I_A^*) + c'(I_B^*)(\bar{I} - I_B^*).$$

This means that the mechanism must specify a minimum level of investment or quality that is high enough to ensure that the total expected profit under the mechanism is higher than the total expected profit without a mechanism plus the total reward paid by the mechanism.

To find such a level of $\bar{I}$, we can use the following algorithm:

- Start with an initial guess of $\bar{I}$, such as $\bar{I} = (I_A^* + I_B^*)/2$.
- Check if the inequality $q(\bar{I})R - 2c(\bar{I}) \geq q^*R - c(I_A^*) - c(I_B^*) - 2R + c'(I_A^*)(\bar{I} - I_A^*) + c'(I_B^*)(\bar{I} - I_B^*)$ holds. If it does, then stop and return $\bar{I}$ as the solution. If it does not, then increase $\bar{I}$ by a small amount and repeat the check.
- Continue this process until the inequality holds or until $\bar{I}$ reaches an upper bound, such as $\bar{I} = R/c'(0)$.
- If the algorithm finds a solution for $\bar{I}$, then we have shown that there exists a mechanism that can satisfy both the incentive compatibility and the individual rationality conditions. The mechanism specifies $\bar{I}$ as the minimum level of investment or quality, $P = q(\bar{I})R/2 + R - c'(I)(\bar{I} - I)$ as the penalty for non-compliance, and R as the reward for compliance.
- By using such a mechanism, both firms can increase their investment levels from I_A^* and I_B^* to $\bar{I}$, which reduces the probability of failure and increases the expected revenue for both parties. The mechanism also reduces the fragility of the supply network, as it creates a buffer against small negative shocks to relationship strength. Therefore, a commitment device or a third-party intermediary or a platform can help strengthen the networks.

QED.