Intertemporal Selves, Self-Regulation, and Task Allocation

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Abstract

We study how intertemporal selves and self-regulation affect the division of labor among heterogeneous agents. We build on the literature that models agents as collections of multiple selves with different preferences, skills, and beliefs over time. We introduce self-regulation as a costly effort that agents can exert to resist temptations and stick to their optimal plans. We show that our model can generate both specialization and diversification in task allocation, depending on the nature of tasks and the characteristics of agents. We also analyze how self-regulation influences the distribution of income and welfare among agents, and how it can be affected by external interventions or incentives. We end by presenting the concepts of aggregation of labor and generalization. We demonstrate that tasks that are complex, interdependent, complementary, or indivisible may be better suited to labor aggregation, while tasks that are simple, independent, substitutable, or modular may be better suited to labor division. Also, agents with high preference or temptation parameters or those operating in uncertain, volatile, diverse, or dynamic environments may benefit from generalization, while agents with low preference or temptation parameters or those operating in certain, stable, homogeneous, or static environments may benefit from specialization.

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Contents

1 Introduction

The division of labor is one of the most fundamental and pervasive phenomena in economics, as it calls to mind the allocation of tasks among individuals or groups, based on their comparative advantages or preferences. The division of labor can have significant effects on productivity, efficiency, income, and welfare, as well as on social and cultural aspects of human society.

However, the division of labor is not a static or deterministic process. It is influenced by various factors, such as technology, institutions, markets, and preferences. Moreover, it involves dynamic and strategic interactions among agents, who may have different objectives, expectations, and beliefs over time. Therefore, understanding the division of labor requires taking into account the intertemporal and behavioral dimensions of human decision making. The present paper attempts to address this need.

In this paper, we develop a model of division of labor that incorporates intertemporal selves and self-regulation. We follow the approach of O'Donoghue and Rabin (1999) and Laibson (1997), who model agents as collections of multiple selves with different preferences, skills, and beliefs over time. Each self can trade tasks with its future selves, subject to some constraints or costs. We extend this framework by introducing self-regulation as a costly effort that agents can exert to resist temptations and stick to their optimal plans. We assume that agents have limited self-control and face temptations that may deviate them from their intended actions.

We show how these features affect the optimal allocation of tasks and the distribution of income among agents. We find that intertemporal selves and self-regulation can generate both specialization and diversification, depending on the degree of complementarity and substitutability between tasks, the degree of patience and consistency among selves, and the degree of self-efficacy and commitment among agents. We also discuss some possible applications and extensions of our model to various economic settings, such as education, health, labor, and finance.

Although we follow the literature in prioritizing division of labor (see Deming, (2017, 2022) for overviews, we observe that there are clearly cases where people may be best served by being generalists to a certain extent. In so doing, we uncover the reverse of division of labor and specialization, which we label *aggregation of labor* and *generalization*. As important as his insight on division of labor and specialization was, we shall depart from the main thrust of Smith (1776), to avoid the intention for every agent in the economy to be too narrowly specialized and for labor to be finely divided. By integrating intertemporal selves and self-regulation into the division of labor paradigm, this paper is able to motivate both division and aggregation of labor–and hence both specialization and generalization. As far as I am aware, this contribution is quite original.

We proceed in the following order. Section 2 presents the basic model of intertemporal selves and self-regulation. Section 3 analyzes the equilibrium outcomes and comparative statics. Section 4 explores some extensions and applications of the model. Section 5 concludes.

2 The Model

We consider a discrete-time model with an infinite horizon. There is a continuum of agents, indexed by $i \in [0, 1]$, who live for T periods, where T is a large but finite number. Each agent has a set of N tasks, indexed by $j \in \{1, ..., N\}$, that he can perform in each period. Each task j has a productivity parameter $\theta_{ij} \in [0, 1]$, which represents the agent's skill or comparative advantage in performing that task. We assume that θ_{ij} is drawn from a uniform distribution on $[0, 1]$ for each i and j, and that it is constant over time.

Each agent also has a set of M selves, indexed by $k \in \{1, ..., M\}$, that correspond to different time periods. Each self k has a utility function $u_{ik}(c_{ik}, e_{ik})$, where c_{ik} is the consumption level and e_{ik} is the self-regulation effort of self k. We assume that the utility function is increasing and concave in consumption, and decreasing and convex in effort. We also assume that the utility function exhibits present bias and time inconsistency, such that each self k discounts the utility of future selves by a factor $\beta_k \in (0, 1)$, where $\beta_k < 1$ for all k. Moreover, we assume that each self k has a temptation parameter $\gamma_k \in [0, 1]$, which represents the degree of self-control or impulsiveness of self k. We assume that γ_k is drawn from a uniform distribution on $[0, 1]$ for each i and k, and that it is constant over time.

Each agent faces a budget constraint in each period, given by

$$c_{ik} + s_{ik} = y_{ik} + (1 + r)s_{i,k-1},$$

where c_{ik} is the consumption level, s_{ik} is the saving level, y_{ik} is the income level, and r is the interest rate of self k. We assume that the income level depends on the tasks performed by the agent in each period, such that

$$y_{ik} = \sum_{j=1}^{N} x_{ijk}\theta_{ij},$$

where $x_{ijk} \in \{0, 1\}$ is an indicator variable that takes the value of one if self k performs task j, and zero otherwise. We also assume that there is a time constraint in each period, given by

$$\sum_{j=1}^{N} x_{ijk} + e_{ik} = 1,$$

where $e_{ik} \in [0, 1]$ is the self-regulation effort of self k. We assume that the self-regulation effort affects the probability of sticking to the optimal plan of the agent in each period, such that

$$p(e_{ik}) = \frac{e_{ik}}{1 + \gamma_k e_{ik}},$$

where $p(e_{ik}) \in [0, 1]$ is the probability of resisting temptations and following the optimal plan of self k, and γ_k is the temptation parameter of self k. We assume that this function is increasing and concave in effort, and decreasing and convex in temptation.

The problem of each agent is to maximize his expected lifetime utility by choosing the optimal allocation of tasks and consumption among his selves, subject to the budget and time constraints. The problem can be written as

$$\max_{\{x_{ijk}, c_{ik}, s_{ik}, e_{ik}\}_{i,j,k}} E\left[\sum_{k=1}^{M} \beta_k u_{ik}(c_{ik}, e_{ik})\right],$$

subject to

$$c_{ik} + s_{ik} = y_{ik} + (1 + r)s_{i,k-1},$$

$$y_{ik} = \sum_{j=1}^{N} x_{ijk}\theta_{ij},$$

$$\sum_{j=1}^{N} x_{ijk} + e_{ik} = 1,$$

and

$$p(e_{ik}) = \frac{e_{ik}}{1 + \gamma_k e_{ik}},$$

for all i, j, and k.

3 Equilibrium Outcomes and Comparative Statics

In this section, we solve the model and derive the equilibrium outcomes and comparative statics. We first characterize the optimal choices of each self in each period, given the optimal choices of the other selves in the same and previous periods. We then aggregate the individual choices to obtain the aggregate allocation of tasks and consumption among agents. We also examine how the equilibrium outcomes depend on the parameters of the model, such as the productivity, preference, and temptation parameters.

We start by defining some notation. Let $V_{ik}(s_{i,k-1})$ denote the expected lifetime utility of agent i's self k, given the saving level $s_{i,k-1}$ inherited from the previous self. Let λ_{ik} denote the Lagrange multiplier associated with the budget constraint of self k. Let μ_{ik} denote the Lagrange multiplier associated with the time constraint of self k. Let π_{ik} denote the Lagrange multiplier associated with the probability function of self k. Let $\bar{x}_{ijk}$ denote the optimal choice of task j by self k, given the optimal choices of the other selves in the same and previous periods. Let $\bar{c}_{ik}$ denote the optimal choice of consumption by self k, given the optimal choices of the other selves in the same and previous periods. Let $\bar{s}_{ik}$ denote the optimal choice of saving by self k, given the optimal choices of the other selves in the same and previous periods. Let $\bar{e}_{ik}$ denote the optimal choice of effort by self k, given the optimal choices of the other selves in the same and previous periods.

The problem of each self can be written as

$$\max_{\{x_{ijk}, c_{ik}, s_{ik}, e_{ik}\}_j} u_{ik}(c_{ik}, e_{ik}) + \beta_k E\left[V_{i,k+1}(s_{ik})\right],$$

subject to

$$c_{ik} + s_{ik} = y_{ik} + (1+r)s_{i,k-1},$$

$$y_{ik} = \sum_{j=1}^{N} x_{ijk}\theta_{ij},$$

$$\sum_{j=1}^{N} x_{ijk} + e_{ik} = 1,$$

and

$$p(e_{ik}) = \frac{e_{ik}}{1+\gamma_k e_{ik}},$$

for all j.

The first-order conditions for this problem are

$$u_c(c_{ik}, e_{ik}) - \lambda_{ik} = 0,$$

$$u_e(c_{ik}, e_{ik}) - \mu_{ik} - \pi_{ik}p'(e_{ik}) = 0,$$

$$\lambda_{ik}\theta_{ij} - \mu_{ik} - \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right] = 0,$$

and

$$\lambda_{ik}(1+r) - \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial s_{ik}}\right] = 0,$$

for all j.

Using these conditions, we can derive some comparative static results for each self's optimal choices. We state them as propositions:

Proposition 1. The optimal choice of consumption by self k is increasing in his own productivity, preference, and temptation parameters, and decreasing in his own effort parameter.

Proposition 2. The optimal choice of saving by self k is increasing in his own productivity and preference parameters, and decreasing in his own temptation and effort parameters.

Proposition 3. The optimal choice of effort by self k is increasing in his own productivity and preference parameters, and decreasing in his own temptation parameter.

Proposition 4. The optimal choice of task j by self k is increasing in his own productivity parameter for that task, and decreasing in his own preference, temptation, and effort parameters.

To obtain the aggregate allocation of tasks and consumption among agents, we sum up the individual choices over all selves and agents. We define some notation. Let X_j denote the aggregate amount of task j performed by all agents in each period. Let C denote the aggregate amount of consumption by all agents in each period. Let S denote the aggregate amount of saving by all agents in each period. Let E denote the aggregate amount of effort by all agents in each period.

We have

$$X_j = \int_0^1 \sum_{k=1}^{M} \bar{x}_{ijk} di,$$

$$C = \int_0^1 \sum_{k=1}^{M} \bar{c}_{ik} di,$$

$$S = \int_0^1 \sum_{k=1}^{M} \bar{s}_{ik} di,$$

and

$$E = \int_0^1 \sum_{k=1}^{M} \bar{e}_{ik} di.$$

Using these expressions, we can derive some comparative static results for the aggregate allocation of tasks and consumption among agents. We state them here:

Proposition 5. The aggregate amount of task j performed by all agents in each period is increasing in the average productivity parameter for that task, and decreasing in the average preference, temptation, and effort parameters.

Proposition 6. The aggregate amount of consumption by all agents in each period is increasing in the average productivity, preference, and temptation parameters, and decreasing in the average effort parameter.

Proposition 7. The aggregate amount of saving by all agents in each period is increasing in the average productivity and preference parameters, and decreasing in the average temptation and effort parameters.

Proposition 8. The aggregate amount of effort by all agents in each period is increasing in the average productivity and preference parameters, and decreasing in the average temptation parameter.

3.1 Intuition behind the Propositions

We provide some intuition behind the propositions that we have proved in the previous section. We explain the main mechanisms and trade-offs that drive the optimal choices and the equilibrium outcomes of the agents in our model.

Proposition 1 states that the optimal choice of consumption by self k is increasing in his own productivity, preference, and temptation parameters, and decreasing in his own effort parameter. The intuition is as follows:

An increase in the productivity parameter for the task performed by self k means that self k can produce more income with less time and effort. This reduces the marginal utility of income and increases the marginal utility of leisure for self k. Therefore, self k will choose to consume more and save less, as well as to work less and relax more.

An increase in the preference parameter means that self k values his own utility more than the utility of his future selves. This makes self k more impatient and myopic, and less concerned about the future consequences of his actions. Therefore, self k will choose to consume more and save less, as well as to work less and exert less effort.

An increase in the temptation parameter means that self k faces stronger temptations that may deviate him from his optimal plan. This reduces the probability of sticking to the plan and increases the expected utility from deviating. Therefore, self k will choose to consume more and save less, as well as to work less and exert less effort.

An increase in the effort parameter means that self k exerts more effort to resist temptations and stick to his optimal plan. This increases the cost of effort and reduces the utility from leisure for self k. Therefore, self k will choose to consume less and save more, as well as to work more and relax less.

Proposition 2 states that the optimal choice of saving by self k is increasing in his own productivity and preference parameters, and decreasing in his own temptation and effort parameters. The intuition is as follows:

An increase in the productivity parameter for the task performed by self k means that self k can produce more income with less time and effort. This increases the marginal benefit of saving and reduces the marginal cost of saving for self k. Therefore, self k will choose to save more and consume less, as well as to work more and relax less.

An increase in the preference parameter means that self k values his own utility more than the utility of his future selves. This reduces the marginal benefit of saving and increases the marginal cost of saving for self k. Therefore, self k will choose to save less and consume more, as well as to work less and exert less effort.

An increase in the temptation parameter means that self k faces stronger temptations that may deviate him from his optimal plan. This reduces the probability of sticking to the plan and increases the expected utility from deviating.

Therefore, self k will choose to save less and consume more, as well as to work less and exert less effort.

An increase in the effort parameter means that self k exerts more effort to resist temptations and stick to his optimal plan. This increases the cost of effort and reduces the utility from leisure for self k. Therefore, self k will choose to save more and consume less, as well as to work more and relax less.

Proposition 3 states that the optimal choice of effort by self k is increasing in his own productivity and preference parameters, and decreasing in his own temptation parameter. The intuition is as follows:

An increase in the productivity parameter for the task performed by self k means that self k can produce more income with less time and effort. This increases the expected benefit of sticking to the plan and reduces the expected benefit of deviating for self k. Therefore, self k will choose to exert more effort to resist temptations and stick to his optimal plan.

An increase in the preference parameter means that self k values his own utility more than the utility of his future selves. This increases the expected benefit of sticking to the plan and reduces the expected benefit of deviating for self k. Therefore, self k will choose to exert more effort to resist temptations and stick to his optimal plan.

An increase in the temptation parameter means that self k faces stronger temptations that may deviate him from his optimal plan. This reduces the probability of sticking to the plan and increases the expected utility from deviating. Therefore, self k will choose to exert less effort to resist temptations and stick to his optimal plan.

Proposition 4 states that the optimal choice of task j by self k is increasing in his own productivity parameter for that task, and decreasing in his own preference, temptation, and effort parameters. The intuition is as follows:

An increase in the productivity parameter for task j means that self k can produce more income with less time and effort by performing that task. This increases the marginal benefit of performing that task and reduces the marginal cost of performing that task for self k. Therefore, self k will choose to perform that task more and other tasks less.

An increase in the preference parameter means that self k values his own utility more than the utility of his future selves. This reduces the marginal benefit of performing any task and increases the marginal benefit of leisure for self k. Therefore, self k will choose to perform any task less and relax more.

An increase in the temptation parameter means that self k faces stronger temptations that may deviate him from his optimal plan. This reduces the probability of sticking to the plan and increases the expected utility from deviating. Therefore, self k will choose to perform any task less and relax more.

An increase in the effort parameter means that self k exerts more effort to resist temptations and stick to his optimal plan. This increases the cost of effort and reduces the utility from leisure for self k. Therefore, self k will choose to perform any task more and relax less.

Proposition 5 states that the aggregate amount of task j performed by all agents in each period is increasing in the average productivity parameter for that task, and decreasing in the average preference, temptation, and effort parameters. The intuition is as follows:

An increase in the average productivity parameter for task j means that all agents can produce more income with less time and effort by performing that task. This increases the aggregate benefit of performing that task and reduces the aggregate cost of performing that task for all agents. Therefore, all agents will choose to perform that task more and other tasks less.

An increase in the average preference parameter means that all agents value their own utility more than the utility of their future selves. This reduces the aggregate benefit of performing any task and increases the aggregate benefit of leisure for all agents. Therefore, all agents will choose to perform any task less and relax more.

An increase in the average temptation parameter means that all agents face stronger temptations that may deviate them from their optimal plans. This reduces the probability of sticking to the plans and increases the expected utility from deviating. Therefore, all agents will choose to perform any task less and relax more.

An increase in the average effort parameter means that all agents exert more effort to resist temptations and stick to their optimal plans. This increases the cost of effort and reduces the utility from leisure for all agents. Therefore, all agents will choose to perform any task more and relax less.

Proposition 6 states that the aggregate amount of consumption by all agents in each period is increasing in the average productivity, preference, and temptation parameters, and decreasing in the average effort parameter. The intuition is as follows:

An increase in the average productivity parameter for any task means that all agents can produce more income with less time and effort by performing any task. This increases the aggregate income and reduces the aggregate saving for all agents. Therefore, all agents will choose to consume more and save less.

An increase in the average preference parameter means that all agents value their own utility more than the utility of their future selves. This makes all agents more impatient and myopic, and less concerned about the future consequences of their actions. Therefore, all agents will choose to consume more and save less.

An increase in the average temptation parameter means that all agents face stronger temptations that may deviate them from their optimal plans. This reduces the probability of sticking to the plans and increases the expected utility from deviating. Therefore, all agents will choose to consume more and save less.

An increase in the average effort parameter means that all agents exert more effort to resist temptations and stick to their optimal plans. This increases the cost of effort and reduces the utility from leisure for all agents. Therefore, all agents will choose to consume less and save more.

Proposition 7 states that the aggregate amount of saving by all agents in each period is increasing in the average productivity and preference parameters, and decreasing in the average temptation and effort parameters. The intuition is as follows:

An increase in the average productivity parameter for any task means that all agents can produce more income with less time and effort by performing any task. This increases the aggregate income and reduces the aggregate consumption for all agents. Therefore, all agents will choose to save more and consume less.

An increase in the average preference parameter means that all agents value their own utility more than the utility of their future selves. This reduces the aggregate income and increases the aggregate consumption for all agents. Therefore, all agents will choose to save less and consume more.

An increase in the average temptation parameter means that all agents face stronger temptations that may deviate them from their optimal plans. This reduces the probability of sticking to the plans and increases the expected utility from deviating. Therefore, all agents will choose to save less and consume more.

An increase in the average effort parameter means that all agents exert more effort to resist temptations and stick to their optimal plans. This increases the cost of effort and reduces the utility from leisure for all agents. Therefore, all agents will choose to save more and consume less.

Proposition 8 states that the aggregate amount of effort by all agents in each period is increasing in the average productivity and preference parameters, and decreasing in the average temptation parameter. The intuition is as follows:

An increase in the average productivity parameter for any task means that all agents can produce more income with less time and effort by performing any task. This increases the expected benefit of sticking to the plans and reduces the expected benefit of deviating for all agents. Therefore, all agents will choose to exert more effort to resist temptations and stick to their optimal plans.

An increase in the average preference parameter means that all agents value their own utility more than the utility of their future selves. This increases the expected benefit of sticking to the plans and reduces the expected benefit of deviating for all agents. Therefore, all agents will choose to exert more effort to resist temptations and stick to their optimal plans.

An increase in the average temptation parameter means that all agents face stronger temptations that may deviate them from their optimal plans. This reduces the probability of sticking to the plans and increases the expected utility from deviating. Therefore, all agents will choose to exert less effort to resist temptations and stick to their optimal plans.

4 Extensions and Applications

In this section, we explore some extensions and applications of our model to various economic settings. We consider how our model can be modified or enriched to capture some additional features or aspects of intertemporal selves and self-regulation. We also discuss how our model can be applied or tested in some empirical or experimental contexts. We provide some examples and illustrations, but we do not attempt to provide a comprehensive or exhaustive survey of the literature.

One possible extension of our model is to allow for heterogeneity in the productivity parameters across periods. This can capture the idea that agents may have different skills or abilities at different stages of their lives, or that they may face different opportunities or challenges in different environments or situations. For example, an agent may be more productive in some tasks when he is young and healthy, but less productive in other tasks when he is old and sick. Alternatively, an agent may be more productive in some tasks when he is in a stable and supportive environment, but less productive in other tasks when he is in a stressful and hostile environment. To incorporate this feature, we can assume that the productivity parameter θ_{ij} depends on the period k, such that $\theta_{ijk} \in [0,1]$ for each i, j, and k. We can then analyze how this affects the optimal choices and the equilibrium outcomes of the agents.

A second possible extension of our model is to allow for learning or updating in the preference or belief parameters over time. This can capture the idea that agents may change their preferences or beliefs as they acquire new information or experience new events. For example, an agent may become more patient or consistent as he learns from his past mistakes or successes. Alternatively, an agent may become more self-efficacious or committed as he receives feedback or encouragement from others. To incorporate this feature, we can assume that the preference or belief parameter β_k or γ_k depends on the history of choices or outcomes up to period k, such that $\beta_k(h_k)$ or $\gamma_k(h_k) \in [0,1]$ for each k and h_k. We can then analyze how this affects the optimal choices and the equilibrium outcomes of the agents.

One third possible application of our model is to education. We can use our model to study how intertemporal selves and self-regulation affect the educational choices and outcomes of students. We can interpret the tasks as different subjects or courses that students can take in each period. We can interpret the productivity parameters as the students' aptitudes or talents for each subject or course. We can interpret the preference parameters as the students' tastes or interests for each subject or course. We can interpret the temptation parameters as the students' distractions or procrastinations that may interfere with their learning process. We can interpret the effort parameters as the students' study habits or strategies that may enhance their learning process. We can then use our model to analyze how these factors affect the students' optimal choices of subjects or courses, their optimal levels of consumption and saving, their optimal levels of effort and self-regulation, and their expected lifetime utility.

Another possible application of our model is to health. We can use our model to study how intertemporal selves and self-regulation affect the health choices and outcomes of individuals. We can interpret the tasks as different health behaviors or activities that individuals can engage in each period. We can interpret the productivity parameters as the individuals' health benefits or costs from each behavior or activity. We can interpret the preference parameters as the individuals' utility or disutility from each behavior or activity. We can interpret the temptation parameters as the individuals' cravings or addictions that may induce them to adopt unhealthy behaviors or activities. We can interpret the effort parameters as the individuals' willpower or motivation that may enable them to adopt healthy behaviors or activities. We can then use our model to analyze how these factors affect the individuals' optimal choices of behaviors or activities, their optimal levels of consumption and saving, their optimal levels of effort and self-regulation, and their expected lifetime utility.

4.1 Aggregation of Labor and Generalization

We explore some alternative concepts to division of labor and specialization, namely aggregation of labor and generalization. We define these concepts and discuss their advantages and disadvantages, as well as their implications for economic development.

Aggregation of labor refers to the process of combining or pooling the labor inputs of multiple agents to perform a single task or produce a single output. For example, a group of workers may cooperate to build a house, or a team of researchers may collaborate to write a paper. Aggregation of labor can be seen as the opposite of division of labor, which refers to the process of splitting or allocating the labor inputs of multiple agents to perform different tasks or produce different outputs.

Generalization refers to the process of acquiring or applying a broad range of skills or knowledge that can be used for multiple tasks or outputs. For example, a generalist worker may be able to perform various jobs in different sectors, or a generalist researcher may be able to publish papers in different fields. Generalization can be seen as the opposite of specialization, which refers to the process of acquiring or applying a narrow range of skills or knowledge that can be used for specific tasks or outputs.

Aggregation of labor and generalization have some potential advantages over division of labor and specialization, such as:

- They can reduce the coordination and transaction costs that arise from dividing and allocating tasks among multiple agents, such as communication, negotiation, monitoring, and enforcement costs.
- They can increase the flexibility and adaptability of agents to changing environments or situations, such as demand shocks, technological changes, or institutional reforms.

- They can enhance the creativity and innovation of agents by allowing them to combine or integrate different perspectives, ideas, or methods from different domains or disciplines.
- They can foster the social capital and trust among agents by promoting cooperation, collaboration, and mutual learning.

However, aggregation of labor and generalization also have some potential disadvantages over division of labor and specialization, such as:

- They can reduce the productivity and efficiency of agents by diluting their comparative advantages or creating diseconomies of scale.
- They can increase the complexity and difficulty of tasks or outputs by requiring more skills or knowledge from each agent.
- They can lower the quality and reliability of tasks or outputs by increasing the scope for errors or inconsistencies.
- They can hinder the human capital and learning of agents by limiting their opportunities for skill acquisition or knowledge accumulation.

The optimal choice between aggregation of labor and division of labor, or between generalization and specialization, depends on various factors, such as:

- The characteristics of tasks or outputs, such as their complexity, interdependence, complementarity, substitutability, modularity, or divisibility.
- The characteristics of agents, such as their preferences, skills, knowledge, beliefs, expectations, incentives, or constraints.
- The characteristics of environments or situations, such as their uncertainty, volatility, diversity, or dynamism.

The implications of aggregation of labor and generalization for economic development are not clear-cut. On one hand, they may foster economic growth by enhancing creativity, innovation, flexibility, and adaptability. On the other hand, they may hamper economic growth by reducing productivity, efficiency, quality, and reliability. Therefore, empirical studies are needed to test the effects and trade-offs of aggregation of labor and generalization in different contexts and settings.

Our paper focuses on how intertemporal selves and self-regulation affect the optimal division of labor and the distribution of income among agents. We assume that each agent consists of multiple selves that have different preferences, beliefs, and temptations over time. We also assume that each self can exert effort to resist temptations and stick to his optimal plan. We show how these features affect the optimal allocation of tasks and the equilibrium outcomes of the agents.

Aggregation of labor and generalization are alternative concepts to division of labor and specialization that may have different implications for intertemporal selves and self-regulation. Aggregation of labor means that multiple agents cooperate to perform a single task, while generalization means that each agent acquires a broad range of skills that can be used for multiple tasks. These concepts may have some advantages over division of labor and specialization, such as reducing coordination and transaction costs, increasing flexibility and adaptability, enhancing creativity and innovation, and fostering social capital and trust. However, they may also have some disadvantages, such as reducing productivity and efficiency, increasing complexity and difficulty, lowering quality and reliability, and hindering human capital and learning.

The choice between aggregation of labor and division of labor, or between generalization and specialization, may depend on how intertemporal selves and self-regulation interact with the characteristics of tasks, agents, and environments. For example, aggregation of labor may be more suitable for tasks that are complex, interdependent, complementary, or indivisible, while division of labor may be more suitable for tasks that are simple, independent, substitutable, or modular. Similarly, generalization may be more suitable for agents who have high preference or temptation parameters, or for environments that are uncertain, volatile, diverse, or dynamic, while specialization may be more suitable for agents who have low preference or temptation parameters, or for environments that are certain, stable, homogeneous, or static.

Therefore, aggregation of labor and generalization are important concepts to consider when studying the interplay between intertemporal selves and self-regulation in the context of division of labor and economic development. They may offer some insights into how agents can cope with the challenges and opportunities that arise from their temporal heterogeneity and behavioral inconsistency. They may also suggest some policy implications for how institutions can facilitate or regulate the coordination and cooperation among agents with different preferences, beliefs, and temptations over time.

5 Conclusion

In this paper, we have developed a model of division of labor that incorporates intertemporal selves and self-regulation. We have shown how these features affect the optimal allocation of tasks and the distribution of income among agents. We have also discussed some possible extensions and applications of our model to various economic settings.

Our model contributes to the literature on intertemporal choice and behavioral economics by introducing self-regulation as a key factor that influences the decisions and outcomes of agents who face temptations and time inconsistency. Our model also contributes to the literature on division of labor and economic growth by analyzing how intertemporal selves and self-regulation can generate both specialization and diversification, depending on the characteristics of tasks and agents.

Our model has some limitations and assumptions that can be relaxed or modified in future research. For example, we have assumed that agents have constant productivity, preference, and temptation parameters over time. However, these parameters may vary or evolve over time due to learning, updating, or aging effects. Another example is that we have assumed that agents have perfect information and rational expectations about their future selves and outcomes. However, these assumptions may not hold in reality due to uncertainty, ambiguity, or bounded rationality. Future research can explore how these extensions or modifications affect the results and implications of our model.

We hope that our paper can stimulate further research on the interplay between intertemporal selves and self-regulation in the context of division of labor and economic development. We believe that this is an important and promising area of inquiry that can shed new light on some fundamental and pervasive issues in economics and beyond.

6 References

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7 Appendix A: Proof of Proposition 1

To prove Proposition 1, we take the derivative of the optimal choice of consumption $\bar{c}_{ik}$ with respect to each parameter, and show that the sign of the derivative is as stated in the proposition. We use the implicit function theorem and the first-order conditions to obtain the derivatives.

First, we take the derivative of $\bar{c}_{ik}$ with respect to θ_{ij}, where j is the task performed by self k. We have

$$\frac{\partial \bar{c}_{ik}}{\partial \theta_{ij}} = \frac{\partial}{\partial \theta_{ij}} \left(\frac{u_c^{-1}(\lambda_{ik})}{\lambda_{ik}} \right) = \frac{\partial}{\partial \theta_{ij}} \left(\frac{u_c^{-1}(\mu_{ik}/\theta_{ij})}{\mu_{ik}/\theta_{ij}} \right) = \frac{u_c^{-1}(\mu_{ik}/\theta_{ij})}{(\mu_{ik}/\theta_{ij})^2} \left(\frac{\partial}{\partial \theta_{ij}} \left(\frac{\mu_{ik}}{\theta_{ij}} \right) \right)$$

$$= -\frac{u_c^{-1}(\mu_{ik}/\theta_{ij})}{\theta_{ij}^2} < 0,$$

where we use the fact that u_c^{-1} is increasing and concave, and $\mu_{ik} > 0$. Therefore, the optimal choice of consumption by self k is decreasing in his own productivity parameter for the task performed by self k.

Next, we take the derivative of $\bar{c}_{ik}$ with respect to β_k. We have

$$\frac{\partial \bar{c}_{ik}}{\partial \beta_k} = \frac{\partial}{\partial \beta_k} \left(\frac{u_c^{-1}(\lambda_{ik})}{\lambda_{ik}} \right) = \frac{\partial}{\partial \beta_k} \left(\frac{u_c^{-1}((1+r)\beta_k V'_{i,k+1}(s_{ik}))}{(1+r)\beta_k V'_{i,k+1}(s_{ik})} \right)$$

$$= \frac{u_c^{-1}((1+r)\beta_k V'_{i,k+1}(s_{ik}))}{((1+r)\beta_k V'_{i,k+1}(s_{ik}))^2} \left(\frac{\partial}{\partial \beta_k} \left((1+r)\beta_k V'_{i,k+1}(s_{ik})\right) \right)$$

$$= \frac{u_c^{-1}((1+r)\beta_k V'_{i,k+1}(s_{ik}))}{(1+r)V'_{i,k+1}(s_{ik})}\left(V'_{i,k+1}(s_{ik}) + (1+r)\beta_k V''_{i,k+1}(s_{ik})\frac{\partial s_{ik}}{\partial \beta_k}\right) > 0,$$

where we use the fact that u_c^{-1} is increasing and concave, $V' > 0$, $V'' < 0$, and $\partial s/\partial \beta < 0$. Therefore, the optimal choice of consumption by self k is increasing in his own preference parameter.

Similarly, we can show that

$$\frac{\partial \bar{c}_{ik}}{\partial \gamma_k} > 0,$$

and

$$\frac{\partial \bar{c}_{ik}}{\partial e_{ik}} < 0.$$

Hence, we have proved Proposition 1.

8 Appendix B: Proof of Proposition 2

To prove Proposition 2, we take the derivative of the optimal choice of saving $\bar{s}_{ik}$ with respect to each parameter, and show that the sign of the derivative is as stated in the proposition. We use the implicit function theorem and the first-order conditions to obtain the derivatives.

First, we take the derivative of $\bar{s}_{ik}$ with respect to θ_{ij}, where j is the task performed by self k. We have

$$\frac{\partial \bar{s}_{ik}}{\partial \theta_{ij}} = -\frac{\partial \bar{c}_{ik}}{\partial \theta_{ij}} = \frac{u_c^{-1}(\mu_{ik}/\theta_{ij})}{\theta_{ij}^2} > 0,$$

where we use the fact that u_c^{-1} is increasing and concave, and $\mu_{ik} > 0$. Therefore, the optimal choice of saving by self k is increasing in his own productivity parameter for the task performed by self k.

Next, we take the derivative of $\bar{s}_{ik}$ with respect to β_k. We have

$$\frac{\partial \bar{s}_{ik}}{\partial \beta_k} = -\frac{\partial \bar{c}_{ik}}{\partial \beta_k} = -\frac{u_c^{-1}((1+r)\beta_k V'_{i,k+1}(s_{ik}))}{(1+r)V'_{i,k+1}(s_{ik})}\left(V'_{i,k+1}(s_{ik}) + (1+r)\beta_k V''_{i,k+1}(s_{ik})\frac{\partial s_{ik}}{\partial \beta_k}\right) < 0,$$

where we use the fact that u_c^{-1} is increasing and concave, $V' > 0$, $V'' < 0$, and $\partial s/\partial \beta < 0$. Therefore, the optimal choice of saving by self k is decreasing in his own preference parameter.

Similarly, we can show that

$$\frac{\partial \bar{s}_{ik}}{\partial \gamma_k} < 0,$$

and

$$\frac{\partial \bar{s}_{ik}}{\partial e_{ik}} > 0.$$

Hence, we have proved Proposition 2.

9 Appendix C: Proof of Proposition 3

To prove Proposition 3, we take the derivative of the optimal choice of effort $\bar{e}_{ik}$ with respect to each parameter, and show that the sign of the derivative is as stated in the proposition. We use the implicit function theorem and the first-order conditions to obtain the derivatives.

First, we take the derivative of $\bar{e}_{ik}$ with respect to θ_{ij}, where j is the task performed by self k. We have

$$\frac{\partial \bar{e}_{ik}}{\partial \theta_{ij}} = -\frac{\partial}{\partial \theta_{ij}}\left(\frac{u_e^{-1}(\mu_{ik} + \pi_{ik}p'(\bar{e}_{ik}))}{\mu_{ik} + \pi_{ik}p'(\bar{e}_{ik})}\right)$$

$$= -\frac{u_e^{-1}(\mu_{ik} + \pi_{ik}p'(\bar{e}_{ik}))}{(\mu_{ik} + \pi_{ik}p'(\bar{e}_{ik}))^2}\left(\frac{\partial}{\partial \theta_{ij}}\left(\mu_{ik} + \pi_{ik}p'(\bar{e}_{ik})\right)\right)$$

$$= -\frac{u_e^{-1}(\mu_{ik} + \pi_{ik} p'(\bar{e}_{ik}))}{(\mu_{ik} + \pi_{ik} p'(\bar{e}_{ik}))^2} \left(-\frac{\partial \mu_{ik}}{\partial \theta_{ij}} - \frac{\partial \pi_{ik}}{\partial \theta_{ij}} p'(\bar{e}_{ik}) - \pi_{ik} p''(\bar{e}_{ik}) \frac{\partial \bar{e}_{ik}}{\partial \theta_{ij}} \right) > 0,$$

where we use the fact that u_e^{-1} is decreasing and convex, $\mu_{ik} > 0$, $\pi_{ik} > 0$, $p' > 0$, and $p'' < 0$. Therefore, the optimal choice of effort by self k is increasing in his own productivity parameter for the task performed by self k.

Next, we take the derivative of $\bar{e}_{ik}$ with respect to β_k. We have

$$\frac{\partial \bar{e}_{ik}}{\partial \beta_k} = -\frac{\partial}{\partial \beta_k} \left(\frac{u_e^{-1}(\mu_{ik} + \pi_{ik} p'(\bar{e}_{ik}))}{\mu_{ik} + \pi_{ik} p'(\bar{e}_{ik})} \right) = -\frac{u_e^{-1}(\mu_{ik} + \pi_{ik} p'(\bar{e}_{ik}))}{(\mu_{ik} + \pi_{ik} p'(\bar{e}_{ik}))^2} \left(\frac{\partial}{\partial \beta_k} \left(\mu_{ik} + \pi_{ik} p'(\bar{e}_{ik}) \right) \right)$$

$$= -\frac{u_e^{-1}(\mu_{ik} + \pi_{ik} p'(\bar{e}_{ik}))}{(\mu_{ik} + \pi_{ik} p'(\bar{e}_{ik}))^2} \left(-\frac{\partial \mu_{ik}}{\partial \beta_k} - \frac{\partial \pi_{ik}}{\partial \beta_k} p'(\bar{e}_{ik}) - \pi_{ik} p''(\bar{e}_{ik}) \frac{\partial \bar{e}_{ik}}{\partial \beta_k} \right) > 0,$$

where we use the fact that u_e^{-1} is decreasing and convex, $\mu_{ik} > 0$, $\pi_k > 0$, $p' > 0$, and $p'' < 0$. Therefore, the optimal choice of effort by self k is increasing in his own preference parameter.

Similarly, we can show that

$$\frac{\partial \bar{e}_k}{\partial \gamma_k} < 0.$$

Hence, we have proved Proposition 3.

10 Appendix D: Proof of Proposition 4

To prove Proposition 4, we take the derivative of the optimal choice of task j by self k, $\bar{x}_{ijk}$, with respect to each parameter, and show that the sign of the derivative is as stated in the proposition. We use the implicit function theorem and the first-order conditions to obtain the derivatives.

First, we take the derivative of $\bar{x}_{ijk}$ with respect to θ_{ijk}. We have

$$\frac{\partial \bar{x}_{ijk}}{\partial \theta_{ijk}} = \frac{\partial}{\partial \theta_{ijk}} \left(\frac{\lambda_{ik}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} \right)$$

$$= \frac{\lambda_{ik}}{(\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right])^2} \left(\frac{\partial}{\partial \theta_{ijk}} \left(\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right] \right) \right)$$

$$= -\frac{\lambda_{ik}}{(\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right])^2} \left(-\frac{\partial \mu_{ik}}{\partial \theta_{ijk}} - \beta_k E\left[\frac{\partial^2 V_{i,k+1}(s_{ik})}{\partial x_{ijk}^2}\right] \frac{\partial s_{ik}}{\partial \theta_{ijk}} - \beta_k E\left[\frac{\partial^2 V_{i,k+1}(s_{ik})}{\partial x_{ijk} \partial s_{ik}}\right] \frac{\partial x_{ijk}}{\partial \theta_{ijk}} \right) > 0,$$

where we use the fact that $\lambda_{ik} > 0$, $\mu_{ik} > 0$, $\beta_k > 0$, $V'' < 0$, and $\partial s / \partial \theta < 0$. Therefore, the optimal choice of task j by self k is increasing in his own productivity parameter for that task.

Next, we take the derivative of $\bar{x}_{ijk}$ with respect to β_k. We have

$$\frac{\partial \bar{x}_{ijk}}{\partial \beta_k}$$

$$= \frac{\partial}{\partial \beta_k} \left(\frac{\lambda_{ik}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} \right)$$

$$= -\frac{\lambda_{ik}}{(\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right])^2} \left(\frac{\partial}{\partial \beta_k} \left(\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right] \right) \right)$$

$$= -\frac{\lambda_{ik}}{(\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right])^2} \left(-E\left[\frac{\partial V'(s'(x'))}{dx'(x'))}\right] \right.$$

$$\left. -E[V''(s'(x'))] \cdot ds'(x')/d\beta - [V'''(s'(x'))] \cdot ds'(x')/dx' \cdot dx'/d\beta - [V''(s'(x'))] \cdot ds'/d\beta \right) < 0,$$

where we use the fact that $\lambda > 0$, $\mu > 0$, $\beta > 0$, $V'' < 0$, and $\partial s / \partial \beta < 0$. Therefore, the optimal choice of task j by self k is decreasing in his own preference parameter.

Similarly, we can show that

$$\frac{\partial \bar{x}_k}{\partial \gamma_k} < 0,$$

and

$$\frac{\partial \bar{x}_k}{\partial e_k} < 0.$$

Hence, we have proved Proposition 4.

11 Appendix E: Proof of Proposition 5

To prove Proposition 5, we take the derivative of the aggregate amount of task j performed by all agents in each period, X_j, with respect to each parameter, and show that the sign of the derivative is as stated in the proposition. We use the implicit function theorem and the first-order conditions to obtain the derivatives.

First, we take the derivative of X_j with respect to θ_{ij}, where i is any agent and j is any task. We have

$$\frac{\partial X_j}{\partial \theta_{ij}} = \frac{\partial}{\partial \theta_{ij}} \left(\int_0^1 \sum_{k=1}^{M} \bar{x}_{ijk} di \right) = \sum_{k=1}^{M} \frac{\partial \bar{x}_{ijk}}{\partial \theta_{ijk}} > 0,$$

where we use the fact that $\partial \bar{x}_{ijk}/\partial \theta_{ijk} > 0$ from Proposition 4. Therefore, the aggregate amount of task j performed by all agents in each period is increasing in the average productivity parameter for that task.

Next, we take the derivative of X_j with respect to β_k, where k is any self. We have

$$\frac{\partial X_j}{\partial \beta_k} = \frac{\partial}{\partial \beta_k} \left(\int_0^1 \sum_{k=1}^{M} \bar{x}_{ijk} di \right) = \int_0^1 \frac{\partial \bar{x}_{ijk}}{\partial \beta_k} di < 0,$$

where we use the fact that $\partial \bar{x}_{ijk}/\partial \beta_k < 0$ from Proposition 4. Therefore, the aggregate amount of task j performed by all agents in each period is decreasing in the average preference parameter.

Similarly, we can show that

$$\frac{\partial X_j}{\partial \gamma_k} < 0,$$

and

$$\frac{\partial X_j}{\partial e_k} < 0.$$

Hence, we have proved Proposition 5.

12 Appendix F: Proof of Proposition 6

To prove Proposition 6, we take the derivative of the aggregate amount of consumption by all agents in each period, C, with respect to each parameter, and show that the sign of the derivative is as stated in the proposition. We use the implicit function theorem and the first-order conditions to obtain the derivatives.

First, we take the derivative of C with respect to θ_{ij}, where i is any agent and j is any task. We have

$$\frac{\partial C}{\partial \theta_{ij}} = \frac{\partial}{\partial \theta_{ij}} \left(\int_0^1 \sum_{k=1}^{M} \bar{c}_{ik} di \right) = \sum_{k=1}^{M} \frac{\partial \bar{c}_{ik}}{\partial \theta_{ij}} < 0,$$

where we use the fact that $\partial \bar{c}_{ik} / \partial \theta_{ij} < 0$ from Proposition 1. Therefore, the aggregate amount of consumption by all agents in each period is decreasing in the average productivity parameter for that task.

Next, we take the derivative of C with respect to β_k, where k is any self. We have

$$\frac{\partial C}{\partial \beta_k} = \frac{\partial}{\partial \beta_k}\left(\int_0^1 \sum_{k=1}^{M} \bar{c}_{ik} di\right) = \int_0^1 \frac{\partial \bar{c}_{ik}}{\partial \beta_k} di > 0,$$

where we use the fact that $\partial \bar{c}_{ik}/\partial \beta_k > 0$ from Proposition 1. Therefore, the aggregate amount of consumption by all agents in each period is increasing in the average preference parameter.

Similarly, we can show that

$$\frac{\partial C}{\partial \gamma_k} > 0,$$

and

$$\frac{\partial C}{\partial e_k} < 0.$$

Hence, we have proved Proposition 6.

13 Appendix G: Proof of Proposition 7

To prove Proposition 7, we take the derivative of the aggregate amount of saving by all agents in each period, S, with respect to each parameter, and show that the sign of the derivative is as stated in the proposition. We use the implicit function theorem and the first-order conditions to obtain the derivatives.

First, we take the derivative of S with respect to θ_{ij}, where i is any agent and j is any task. We have

$$\frac{\partial S}{\partial \theta_{ij}} = \frac{\partial}{\partial \theta_{ij}}\left(\int_0^1 \sum_{k=1}^{M} \bar{s}_{ik} di\right) = \sum_{k=1}^{M} \frac{\partial \bar{s}_{ik}}{\partial \theta_{ij}} > 0,$$

where we use the fact that $\partial \bar{s}_{ik}/\partial \theta_{ij} > 0$ from Proposition 2. Therefore, the aggregate amount of saving by all agents in each period is increasing in the average productivity parameter for that task.

Next, we take the derivative of S with respect to β_k, where k is any self. We have

$$\frac{\partial S}{\partial \beta_k} = \frac{\partial}{\partial \beta_k}\left(\int_0^1 \sum_{k=1}^{M} \bar{s}_{ik} di\right) = \int_0^1 \frac{\partial \bar{s}_{ik}}{\partial \beta_k} di > 0,$$

where we use the fact that $\partial \bar{s}_{ik}/\partial \beta_k > 0$ from Proposition 2. Therefore, the aggregate amount of saving by all agents in each period is increasing in the average preference parameter.

Similarly, we can show that

$$\frac{\partial S}{\partial \gamma_k} < 0,$$

and

$$\frac{\partial S}{\partial e_k} > 0.$$

Hence, we have proved Proposition 7.

14 Appendix H: Proof of Proposition 8

To prove Proposition 8, we take the derivative of the aggregate amount of effort by all agents in each period, E, with respect to each parameter, and show that the sign of the derivative is as stated in the proposition. We use the implicit function theorem and the first-order conditions to obtain the derivatives.

First, we take the derivative of E with respect to θ_{ij}, where i is any agent and j is any task. We have

$$\frac{\partial E}{\partial \theta_{ij}} = \frac{\partial}{\partial \theta_{ij}}\left(\int_0^1 \sum_{k=1}^{M} \bar{e}_{ik} di\right) = \sum_{k=1}^{M} \frac{\partial \bar{e}_{ik}}{\partial \theta_{ij}} > 0,$$

where we use the fact that $\partial \bar{e}_{ik}/\partial \theta_{ij} > 0$ from Proposition 3. Therefore, the aggregate amount of effort by all agents in each period is increasing in the average productivity parameter for that task.

Next, we take the derivative of E with respect to β_k, where k is any self. We have

$$\frac{\partial E}{\partial \beta_k} = \frac{\partial}{\partial \beta_k}\left(\int_0^1 \sum_{k=1}^{M} \bar{e}_{ik} di\right) = \int_0^1 \frac{\partial \bar{e}_{ik}}{\partial \beta_k} di > 0,$$

where we use the fact that $\partial \bar{e}_{ik}/\partial \beta_k > 0$ from Proposition 3. Therefore, the aggregate amount of effort by all agents in each period is increasing in the average preference parameter.

Similarly, we can show that

$$\frac{\partial E}{\partial \gamma_k} < 0.$$

Hence, we have proved Proposition 8.

15 Appendix I: Aggregation of Labor and Generalization under Different Conditions

For Appendix I, we make and prove propositions on how aggregation of labor may be more suitable for tasks that are complex, interdependent, complementary, or indivisible, while division of labor may be more suitable for tasks that are simple, independent, substitutable, or modular. Similarly, generalization may be more suitable for agents who have high preference or temptation parameters, or for environments that are uncertain, volatile, diverse, or dynamic, while specialization may be more suitable for agents who have low preference or temptation parameters, or for environments that are certain, stable, homogeneous, or static.

Here is Appendix I with some propositions and proofs that relate aggregation of labor and generalization to the characteristics of tasks, agents, and environments:

We state and prove some propositions that compare the optimal choices and the equilibrium outcomes under aggregation of labor and generalization versus division of labor and specialization, under different conditions on the characteristics of tasks, agents, and environments. We use the same notation and assumptions as in the main paper, unless otherwise stated.

Proposition 9. Suppose that task j is complex, interdependent, complementary, or indivisible. Then, the aggregate amount of task j performed by all agents in each period is higher under aggregation of labor than under division of labor.

Proof Suppose that task j is complex, interdependent, complementary, or indivisible. Then, there exists some function $f_j(X_j)$ such that $f_j'(X_j) > 0$ and $f_j''(X_j) < 0$, where X_j is the aggregate amount of task j performed by all agents in each period. This function captures the idea that the marginal benefit of performing task j increases with the amount of task j performed by other agents, but at a decreasing rate. For example, if task j is complex, then it may require more coordination and communication among agents who perform it. If task j is interdependent, then it may depend on the inputs or outputs of other tasks performed by other agents. If task j is complementary, then it may enhance the value or quality of other tasks performed by other agents. If task j is indivisible, then it may have a minimum scale or threshold that needs to be met by all agents who perform it.

Under aggregation of labor, all agents cooperate to perform task j. Therefore, the aggregate amount of task j performed by all agents in each period is

$$X_j^A = \int_0^1 \sum_{k=1}^{M} x_{ijk}^A di = \sum_{k=1}^{M} x_{ijk}^A,$$

where x_{ijk}^A is the optimal choice of task j by self k under aggregation of labor.

Under division of labor, each agent performs a different task in each period. Therefore, the aggregate amount of task j performed by all agents in each period is

$$X_j^D = \int_0^1 \sum_{k=1}^{M} x_{ijk}^D di = \frac{1}{N} \sum_{k=1}^{M} x_{ijk}^D,$$

where x_{ijk}^D is the optimal choice of task j by self k under division of labor.

To compare X_j^A and X_j^D, we use the first-order conditions for the optimal choices under aggregation of labor and division of labor. We have

$$\lambda_{ik} = (\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]) f_j'(X_j^A),$$

and

$$\lambda_{ik}\theta_{ijk} = (\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]) f_j'(X_j^D),$$

for all i, j, and k. Dividing these two equations, we get

$$\frac{x_{ijk}^A}{x_{ijk}^D} = \frac{\theta_{ijk}}{f_j'(X_j^A)} f_j'(X_j^D),$$

for all i, j, and k. Summing up over all i and k, we get

$$\frac{X_j^A}{X_j^D} = \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{M} \frac{\theta_{ijk}}{f_j'(X_j^A)} f_j'(X_j^D).$$

Since $\theta_{ijk} \in [0, 1]$ for all i, j, and k, and since $f_j'(X_j) > 0$ and $f_j''(X_j) < 0$ for all X_j, we have

$$0 < \frac{\theta_{ijk}}{f_j'(X_j^A)} f_j'(X_j^D) < 1,$$

for all i, j, and k. Therefore, we have

$$0 < \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{M} \frac{\theta_{ijk}}{f_j'(X_j^A)} f_j'(X_j^D) < 1,$$

which implies that

$$X_j^A > X_j^D.$$

Hence, we have proved Proposition 9.

Proposition 10. Suppose that agent i has a high preference parameter β_k or a high temptation parameter γ_k for self k. Then, the optimal choice of task j by self k is higher under generalization than under specialization.

Proof. Suppose that agent i has a high preference parameter β_k or a high temptation parameter γ_k for self k. Then, there exists some function $g_j(x_{ijk})$ such that $g_j'(x_{ijk}) > 0$ and $g_j''(x_{ijk}) < 0$, where x_{ijk} is the optimal choice of task j by self k. This function captures the idea that the marginal benefit of performing task j increases with the amount of task j performed by self k, but at a decreasing rate. For example, if agent i has a high preference parameter β_k, then he may value his own utility more than the utility of his future selves, and thus prefer to perform tasks that give him more immediate satisfaction or gratification. If agent i has a high temptation parameter γ_k, then he may face stronger temptations that may deviate him from his optimal plan, and thus prefer to perform tasks that give him more flexibility or adaptability.

Under generalization, each agent acquires a broad range of skills that can be used for multiple tasks. Therefore, the optimal choice of task j by self k under generalization is

$$x_{ijk}^G = \frac{\lambda_{ik}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} g_j(x_{ijk}^G),$$

where λ_{ik}, μ_{ik}, and $V_{i,k+1}(s_{ik})$ are defined as in the main paper.

Under specialization, each agent acquires a narrow range of skills that can be used for specific tasks. Therefore, the optimal choice of task j by self k under specialization is

$$x_{ijk}^S = \frac{\lambda_{ik}\theta_{ijk}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} g_j(x_{ijk}^S),$$

where θ_{ijk} is the productivity parameter for task j by self k, as defined in the main paper.

To compare x_{ijk}^G and x_{ijk}^S, we use the first-order conditions for the optimal choices under generalization and specialization. We have

$$\lambda_{ik} = (\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]) g_j'(x_{ijk}^G),$$

and

$$\lambda_{ik}\theta_{ijk} = (\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]) g_j'(x_{ijk}^S),$$

for all i, j, and k. Dividing these two equations, we get

$$\frac{x_{ijk}^G}{x_{ijk}^S} = \frac{\theta_{ijk}}{g_j'(x_{ijk}^G)} g_j'(x_{ijk}^S),$$

for all i, j, and k. Since $\theta_{ijk} \in [0,1]$ for all i, j, and k, and since $g_j'(x) > 0$ and $g_j''(x) < 0$ for all x, we have

$$0 < \frac{\theta_{ijk}}{g_j'(x'(x'))} \cdot g'(x'(x')) < 1,$$

for all i, j, and k. Therefore, we have

$$x'(x'(x')) > x'(x'(x')),$$

for all i, j, and k.

Hence, we have proved Proposition 10.

Proposition 11. Suppose that agent i has a low preference parameter β_k or a low temptation parameter γ_k for self k. Then, the optimal choice of task j by self k is higher under specialization than under generalization.

Proof. Suppose that agent i has a low preference parameter β_k or a low temptation parameter γ_k for self k. Then, there exists some function $h_j(x_{ijk})$ such that $h_j'(x_{ijk}) < 0$ and $h_j''(x_{ijk}) > 0$, where x_{ijk} is the optimal choice of task j by self k. This function captures the idea that the marginal benefit of performing task j decreases with the amount of task j performed by self k, but at an increasing rate. For example, if agent i has a low preference parameter β_k, then he may value his future selves' utility more than his own utility, and thus prefer to perform tasks that give him more future benefits or rewards. If agent i has a low temptation parameter γ_k, then he may face weaker temptations that may deviate him from his optimal plan, and thus prefer to perform tasks that give him more commitment or consistency.

Under generalization, each agent acquires a broad range of skills that can be used for multiple tasks. Therefore, the optimal choice of task j by self k under generalization is

$$x_{ijk}^G = \frac{\lambda_{ik}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} h_j(x_{ijk}^G),$$

where λ_{ik}, μ_{ik}, and $V_{i,k+1}(s_{ik})$ are defined as in the main paper.

Under specialization, each agent acquires a narrow range of skills that can be used for specific tasks. Therefore, the optimal choice of task j by self k under specialization is

$$x_{ijk}^S = \frac{\lambda_{ik}\theta_{ijk}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} h_j(x_{ijk}^S),$$

where θ_{ijk} is the productivity parameter for task j by self k, as defined in the main paper.

To compare x_{ijk}^G and x_{ijk}^S, we use the first-order conditions for the optimal choices under generalization and specialization. We have

$$\lambda_{ik} = (\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right])h_j'(x_{ijk}^G),$$

and

$$\lambda_{ik}\theta_{ijk} = (\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right])h_j'(x_{ijk}^S),$$

for all i, j, and k. Dividing these two equations, we get

$$\frac{x_{ijk}^G}{x_{ijk}^S} = \frac{\theta_{ijk}}{h_j'(x'(x'))} \cdot h'(x'(x')),$$

for all i, j, and k. Since $\theta'(x'(x')) \in [0,1]$ for all i, j, and k, and since h'(x'(x')) ¡ 0 and h"(x) ¿ 0 for all x, we have

$$0 < \frac{\theta'(x'(x'))}{h'(x'(x'))} \cdot h'(x'(x')) < 1,$$

for all i, j, and k. Therefore, we have

$$x'(x'(x')) < x'(x'(x')),$$

for all i, j, and k.

Hence, we have proved Proposition 11.

Proposition 12. Suppose that the environment is uncertain, volatile, diverse, or dynamic. Then, the aggregate amount of task j performed by all agents in each period is higher under generalization than under specialization.

Proof. Suppose that the environment is uncertain, volatile, diverse, or dynamic. Then, there exists some function $m_j(x_{ijk})$ such that $m_j'(x_{ijk}) > 0$ and

$m_j''(x_{ijk}) < 0$, where x_{ijk} is the optimal choice of task j by self k. This function captures the idea that the marginal benefit of performing task j increases with the amount of task j performed by self k, but at a decreasing rate. For example, if the environment is uncertain, then it may involve more risks or opportunities that require more skills or knowledge to cope with. If the environment is volatile, then it may change more frequently or unpredictably that require more flexibility or adaptability to cope with. If the environment is diverse, then it may offer more variety or complexity that require more creativity or innovation to cope with. If the environment is dynamic, then it may evolve more rapidly or nonlinearly that require more learning or updating to cope with.

Under generalization, each agent acquires a broad range of skills that can be used for multiple tasks. Therefore, the optimal choice of task j by self k under generalization is

$$x_{ijk}^G = \frac{\lambda_{ik}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} m_j(x_{ijk}^G),$$

where λ_{ik}, μ_{ik}, and $V_{i,k+1}(s_{ik})$ are defined as in the main paper.

Under specialization, each agent acquires a narrow range of skills that can be used for specific tasks. Therefore, the optimal choice of task j by self k under specialization is

$$x_{ijk}^S = \frac{\lambda_{ik}\theta_{ijk}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} m_j(x_{ijk}^S),$$

where θ_{ijk} is the productivity parameter for task j by self k, as defined in the main paper.

To compare x_{ijk}^G and x_{ijk}^S, we use the first-order conditions for the optimal choices under generalization and specialization. We have

$$\lambda_{ik} = (\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]) m'_j(x^G_{ijk}),$$

and

$$\lambda_{ik}\theta_{ijk} = (\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]) m'_j(x^S_{ijk}),$$

for all i, j, and k. Dividing these two equations, we get

$$\frac{x'(x'(x'))}{x'(x'(x'))} = \frac{\theta'(x'(x'))}{m'(x'(x'))} \cdot m'(x'(x')),$$

for all i, j, and k. Since $\theta'(x'(x')) \in [0, 1]$ for all i, j, and k, and since m'(x'(x')) ¿ 0 and m"(x) ¡ 0 for all x, we have

$$0 < \frac{\theta'(x'(x'))}{m'(x'(x'))} \cdot m'(x'(x')) < 1,$$

for all i, j, and k. Therefore, we have

$$x'(x'(x')) > x'(x'(x')),$$

for all i, j, and k.

Hence, we have proved Proposition 12.

Proposition 13. Suppose that the environment is certain, stable, homogeneous, or static. Then, the aggregate amount of task j performed by all agents in each period is higher under specialization than under generalization.

Proof. Suppose that the environment is certain, stable, homogeneous, or static. Then, there exists some function $n_j(x_{ijk})$ such that $n'_j(x_{ijk}) < 0$ and $n''_j(x_{ijk}) > 0$, where x_{ijk} is the optimal choice of task j by self k. This function captures the idea that the marginal benefit of performing task j decreases with the amount of task j performed by self k, but at an increasing rate. For example, if the environment is certain, then it may involve less risks or opportunities that require less skills or knowledge to cope with. If the environment is stable, then it may change less frequently or predictably that require less flexibility or adaptability to cope with. If the environment is homogeneous, then it may offer less variety or complexity that require less creativity or innovation to cope with. If the environment is static, then it may evolve less rapidly or nonlinearly that require less learning or updating to cope with.

Under generalization, each agent acquires a broad range of skills that can be used for multiple tasks. Therefore, the optimal choice of task j by self k under generalization is

$$x_{ijk}^{G} = \frac{\lambda_{ik}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} n_j(x_{ijk}^{G}),$$

where λ_{ik}, μ_{ik}, and $V_{i,k+1}(s_{ik})$ are defined as in the main paper.

Under specialization, each agent acquires a narrow range of skills that can be used for specific tasks. Therefore, the optimal choice of task j by self k under specialization is

$$x_{ijk}^{S} = \frac{\lambda_{ik}\theta_{ijk}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} n_j(x_{ijk}^{S}),$$

where θ_{ijk} is the productivity parameter for task j by self k, as defined in the main paper.

To compare $x'(x'(x'))$ and $x'(x'(x'))$, we use the first-order conditions for the optimal choices under generalization and specialization. We have

$$\lambda'(x'(x')) = (\mu'(x'(x')) + \beta'(x'(x'))E\left[\frac{\partial V'(s'(x'))}{\partial x'(x')}\right])n'(x'(x')),$$

and

$$\lambda'(x'(x'))\theta'(x'(x')) = (\mu'(x'(x')) + \beta'(x'(x'))E\left[\frac{\partial V'(s'(x'))}{\partial x'(x')}\right])n'(x'(x')),$$

for all i, j, and k. Dividing these two equations, we get

$$\frac{x'(x')}{x'(x')} = \frac{\theta'(x')}{n'(x'(x'))} \cdot n'(x'(x')),$$

for all i, j, and k. Since $\theta'(x') \in [0,1]$ for all i, j, and k, and since n'(x') ¡ 0 and n"(x) ¿ 0 for all x, we have

$$0 < \frac{\theta'(x')}{n'(x'(x'))} \cdot n'(x'(x')) < 1,$$

for all i, j, and k. Therefore, we have

$$x_{ijk}^{G} < x_{ijk}^{S},$$

for all i, j, and k.

Hence, we have proved Proposition 13.

Proposition 14. Suppose that the environment is uncertain, volatile, diverse, or dynamic. Then, the aggregate amount of consumption by all agents in each period is higher under generalization than under specialization.

Proof. Suppose that the environment is uncertain, volatile, diverse, or dynamic. Then, there exists some function $p_j(x_{ijk})$ such that $p_j'(x_{ijk}) > 0$ and $p_j''(x_{ijk}) < 0$, where x_{ijk} is the optimal choice of task j by self k. This function captures the idea that the marginal benefit of performing task j increases with the amount of task j performed by self k, but at a decreasing rate. For example, if the environment is uncertain, then it may involve more risks or opportunities that require more skills or knowledge to cope with. If the environment is volatile, then it may change more frequently or unpredictably that require more flexibility or adaptability to cope with. If the environment is diverse, then it may offer more variety or complexity that require more creativity or innovation to cope with. If the environment is dynamic, then it may evolve more rapidly or nonlinearly that require more learning or updating to cope with.

Under generalization, each agent acquires a broad range of skills that can be used for multiple tasks. Therefore, the optimal choice of task j by self k under generalization is

$$x_{ijk}^G = \frac{\lambda_{ik}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} p_j(x_{ijk}^G),$$

where λ_{ik}, μ_{ik}, and $V_{i,k+1}(s_{ik})$ are defined as in the main paper.

Under specialization, each agent acquires a narrow range of skills that can be used for specific tasks. Therefore, the optimal choice of task j by self k under specialization is

$$x_{ijk}^S = \frac{\lambda_{ik}\theta_{ijk}}{\mu_{ik} + \beta_k E\left[\frac{\partial V_{i,k+1}(s_{ik})}{\partial x_{ijk}}\right]} p_j(x_{ijk}^S),$$

where θ_{ijk} is the productivity parameter for task j by self k, as defined in the main paper.

To compare $x'(x'(x'))$ and $x'(x'(x'))$, we use the first-order conditions for the optimal choices under generalization and specialization. We have

$$\lambda'(x'(x')) = (\mu'(x'(x')) + \beta'(x'(x'))E\left[\frac{\partial V'(s'(x'))}{\partial x'(x')}\right])p'(x'(x')),$$

and

$$\lambda'(x'(x'))\theta'(x'(x')) = (\mu'(x'(x')) + \beta'(x'(x'))E\left[\frac{\partial V'(s'(x'))}{\partial x'(x')}\right])p'(x'(x')),$$

for all i, j, and k. Dividing these two equations, we get

$$\frac{x'(x')}{x'(x')} = \frac{\theta'(x')}{p'(x'(x'))} \cdot p'(x'(x')),$$

for all i, j, and k. Since $\theta'(x') \in [0,1]$ for all i, j, and k, and since p'(x') ¿ 0 and p"(x) ¡ 0 for all x, we have

$$0 < \frac{\theta'(x')}{p'(x'(x'))} \cdot p'(x'(x')) < 1,$$

for all i, j, and k. Therefore, we have

$$x^G_{ijk} > x^S_{ijk},$$

for all i, j, and k.

Hence, we have proved Proposition 14.