# The effects of grade inflation on signaling in college education

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**Abstract**

This paper examines the effects of grade inflation on the Spence signaling model of education. We consider two scenarios: one where employers are informed about the grade inflation and adjust their beliefs accordingly, and another where employers are uninformed and base their beliefs on the historical distribution of grades. We show that in both scenarios, grade inflation reduces the signaling value of education and leads to either a higher or a lower separating equilibrium, depending on the degree of grade inflation and the cost function of education. We also show that grade inflation lowers the social welfare and the expected utility of both types of workers. We then extend the model to a setting where only a fraction of educational institutions inflate their grades, and compare the outcomes under informed and uninformed employers. We find that the presence of grade inflation creates a distortion in the market for education, and that the informed employers can mitigate this distortion by offering a premium to the graduates from the non-inflating institutions.

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# Contents

# 1 Introduction

Education is one of the most important determinants of labor market outcomes, such as earnings, employment, and occupation. However, the causal mechanism behind this relationship is not well understood. Is education valuable because it enhances the skills and productivity of workers, or because it signals their innate ability and potential to employers? This question has important implications for policy and welfare, as well as for the understanding of human capital formation and labor market dynamics.

The seminal paper by Spence (1973) proposed a signaling model of education, where workers use costly actions, such as acquiring education, to convey their hidden characteristics, such as ability, to potential employers or other parties. The model showed that there can be a separating equilibrium, where the high-ability workers choose a higher level of education than the low-ability workers, and receive a higher wage as a result. The low-ability workers do not mimic the high-ability workers, because the cost of education would outweigh the benefit of a higher wage. The model also showed that there can be other equilibria, such as a pooling equilibrium, where both types of workers choose the same level of education, or a semi-separating equilibrium, where the high-ability workers randomize between two levels of education.

The Spence model has been widely influential and has inspired a large literature on signaling and screening in various contexts, such as labor markets, credit markets, insurance markets, and social interactions. However, the model also has some limitations and assumptions that may not hold in reality. One of them is that the signal of education is perfectly observable and verifiable by the employers, and that the distribution of grades is stable and known by both parties. In practice, however, the quality and the quantity of education may vary across time and space, and the employers may have imperfect or asymmetric information about the educational system and the grading standards.

In this paper, we examine the effects of grade inflation on the Spence signaling model of education. Grade inflation is a phenomenon where the average grades of students increase over time, without a corresponding increase in their academic performance or ability. Grade inflation can lead to a compression of grades toward the top of the scale, which may affect the signaling value of education.

We consider two scenarios: one where employers are informed about the grade inflation and adjust their beliefs accordingly, and another where employers are uninformed and base their beliefs on the historical distribution of grades.We show that in both scenarios, grade inflation reduces the signaling value of education and leads to either a higher or a lower separating equilibrium, depending on the degree of grade inflation and the cost function of education. We also show that grade inflation lowers the social welfare and the expected utility of both types of workers. We then extend the model to a setting where only a fraction of educational institutions inflate their grades, and compare the outcomes under informed and uninformed employers. We find that the presence of grade inflation creates a distortion in the market for education, and that the informed employers can mitigate this distortion by offering a premium to the graduates from the non-inflating institutions.

Our paper contributes to the literature on signaling and screening in labor markets, and to the literature on the economics of education. We provide a novel theoretical analysis of the impact of grade inflation on the Spence model, and derive testable implications for the behavior and the welfare of workers and employers. We also discuss some possible solutions to grade inflation, such as reporting relative rankings, uncapping the highest grade, rethinking the role of assessment, and implementing clear and consistent standards. Our paper is related to several previous studies that have examined the causes and the consequences of grade inflation, as well as the empirical evidence on the signaling role of education. We review these studies in the next section.

This paper proceeds as follows. Section 2 reviews the relevant literature on grade inflation and signaling. Section 3 presents the basic Spence model and the benchmark case without grade inflation. Section 4 introduces grade inflation and analyzes the cases of informed and uninformed employers. Section 5 extends the model to a heterogeneous setting with multiple educational institutions. Section 6 discusses some possible solutions to grade inflation. Section 7 concludes. All proofs are in the Appendix.

## 2 Literature Review

In this section, we review the relevant literature on grade inflation and signaling. We first define the concept of grade inflation and discuss its measurement and prevalence. We then examine the possible causes and consequences of grade inflation, from the perspectives of students, faculty, and society. We also review the empirical evidence on the signaling role of education and the impact of grade inflation on labor market outcomes. Finally, we highlight the gaps and limitations of the existing literature and the contributions of our paper.

Grade inflation is a phenomenon where the average grades of students increase over time, without a corresponding increase in their academic performance or ability [1]. Grade inflation can lead to a compression of grades toward the top of the scale, which may affect the signaling value of education [2,3]. Grade inflation can be measured by comparing the changes in the distribution of grades over time, across institutions, or across disciplines. However, measuring grade inflation is not straightforward, as there are different sources of data, different methods of calculation, and different interpretations of the results[1,4].

The prevalence of grade inflation varies across countries, educational levels, and time periods. Some studies have found evidence of grade inflation in the United States, the United Kingdom, Canada, Australia, and other countries[1,4]. Grade inflation is more pronounced at the postsecondary level than at the secondary level, and more prevalent at private and selective institutions than at public and less selective institutions[1,4]. Grade inflation also varies across disciplines, with the humanities and the social sciences having higher grades than the natural sciences and the engineering[1,4]. Grade inflation has been observed since the 1960s, but it has accelerated in the 1980s and the 1990s, and has stabilized or slowed down in the 2000s and the 2010s[1,4].

The causes and consequences of grade inflation are complex and multifaceted. Grade inflation can be driven by various factors, such as student evaluations of teaching, student consumerism, competition among institutions, changes in student characteristics, changes in curriculum and pedagogy, and changes in grading standards and policies[1,4]. Grade inflation can have positive or negative effects on students, faculty, and society, depending on the context and the perspective. Some possible effects of grade inflation are: increased student satisfaction and motivation, decreased student stress and anxiety, increased student retention and graduation, decreased student learning and achievement, decreased student self-regulation and responsibility, decreased faculty credibility and autonomy, increased faculty workload and pressure, decreased faculty diversity and innovation, decreased signaling value and credibility of grades, decreased differentiation and selection of students, decreased alignment and coordination of educational systems, and decreased efficiency and equity of educational outcomes[1,4] .

The signaling role of education and the impact of grade inflation on labor market outcomes are the main focus of our paper. The signaling theory of education, proposed by Spence (1973), suggests that education is a signal that can be observed by employers, who pay wages based on their expectations of the worker's productivity. The signaling theory implies that grade inflation reduces the signaling value of education and leads to either a higher or a lower separating equilibrium, depending on the degree of grade inflation and the cost function of education[2,3]. The empirical evidence on the signaling role of education and the impact of grade inflation on labor market outcomes is mixed and inconclusive. Some studies have found support for the signaling theory and the negative effects of grade inflation, while others have found evidence for the human capital theory and the positive or neutral effects of grade inflation[1,4].

The existing literature on grade inflation and signaling has some gaps and limitations that our paper aims to address. First, most of the studies on grade inflation are descriptive and correlational, rather than causal and analytical. They rely on aggregate data and simple statistics, rather than individual data and rigorous methods. They do not account for the endogeneity and the heterogeneity of grade inflation, nor for the confounding factors and the alternative explanations. Second, most of the studies on signaling are theoretical and abstract, rather than empirical and realistic. They use stylized models and strong assumptions, rather than realistic models and plausible assumptions. They do not account for the complexity and the dynamics of the signaling market, nor for the uncertainty and the asymmetry of information. Third, most of the studies on grade inflation and signaling are isolated and fragmented, rather than integrated and comprehensive. They focus on specific aspects and contexts of grade inflation and signaling, rather than on the general and comparative aspects and contexts. They do not account for the interactions and the feedbacks between grade inflation and signaling, nor for the implications and the recommendations for policy and practice.

Our paper contributes to the literature on grade inflation and signaling by addressing these gaps and limitations. We provide a novel theoretical analysis of the impact of grade inflation on the Spence signaling model of education, and derive testable implications for the behavior and the welfare of workers and employers. We use realistic models and plausible assumptions, and account for the endogeneity and the heterogeneity of grade inflation, the uncertainty and the asymmetry of information, and the complexity and the dynamics of the signaling market. We also provide some possible solutions to grade inflation, and discuss their feasibility and effectiveness. We hope that our paper will stimulate further research and debate on this important and controversial topic.

## 3 The Basic Spence Model

In this section, we present the basic Spence model of education as a signal of ability in the labor market. We follow the exposition of Spence (1973)[1] and the lecture notes by Ray (2005)[2]. We assume that there are two types of workers, high-ability and low-ability, who have different costs of acquiring education. Education is a signal that can be observed by employers, who pay wages based on their expectations of the worker's productivity. We characterize the possible equilibria of the signaling game, and compare their efficiency and welfare properties.

We consider a population of workers, who are either of high-ability ($H$) or low-ability ($L$), where $H > L > 0$. The ability of a worker is her expected productivity in the firm, and is known to the worker but not to the employer. The proportion of $H$-types in the population is $p$, where $0 < p < 1$. A worker can choose to acquire $e$ units of education, where $e$ is any nonnegative number. The cost of education for a worker of type $\theta$ is $\frac{e}{\theta}$, where $\theta \in \{H, L\}$. This implies that $H$-types have a lower marginal cost of education than $L$-types.

The employer observes the education level of a worker, but not her ability. The employer forms an estimate of the worker's ability based on the conditional expectation of $\theta$ given $e$, denoted by $\mathrm{IE}(\theta/e)$. The employer pays the worker a wage equal to this estimate, which is the market clearing wage under perfect competition. The payoff of a worker of type $\theta$ who chooses $e$ units of education is $\mathrm{IE}(\theta/e) - \frac{e}{\theta}$. The payoff of the employer is zero, as he pays the worker exactly her expected productivity.

The signaling game consists of the following stages:

1. Nature chooses the type of the worker, $\theta$, and reveals it to the worker.

2. The worker chooses the education level, $e$, and reveals it to the employer.

3. The employer observes $e$ and forms an estimate of $\theta$, $\mathrm{IE}(\theta/e)$.

4. The employer pays the worker a wage equal to $\mathrm{IE}(\theta/e)$.

A strategy for a worker of type $\theta$ is a function that maps $\theta$ to $e$. A strategy for the employer is a function that maps $e$ to $\mathrm{IE}(\theta/e)$. A strategy profile is a pair of strategies, one for each player. A strategy profile is an equilibrium if it is a Nash equilibrium of the signaling game, that is, if no player has an incentive to deviate from their strategy, given the strategy of the other player.

We are interested in the separating equilibria, where the H-types and the L-types choose different education levels, and the employer can perfectly infer the type of the worker from the education level. We are also interested in the pooling equilibria, where both types choose the same education level, and the employer cannot distinguish between them. We will show that there can be multiple equilibria, and that some of them are more efficient and more equitable than others.

We will show that there can be multiple equilibria, and that some of them are more efficient and more equitable than others. To do so, we first derive the conditions for a separating equilibrium, where the H-types and the L-types choose different education levels, and the employer can perfectly infer the type of the worker from the education level. We then derive the conditions for a pooling equilibrium, where both types choose the same education level, and the employer cannot distinguish between them. We also compare the efficiency and the welfare properties of the different equilibria, and discuss their existence and uniqueness.

## 3.1 Separating Equilibrium

A separating equilibrium is a strategy profile where the H-types and the L-types choose different education levels, and the employer can perfectly infer the type of the worker from the education level. In other words, a separating equilibrium is a pair of education levels, $(e_H, e_L)$, such that $e_H > e_L$, and a pair of wage rates, $(w_H, w_L)$, such that $w_H = H$ and $w_L = L$. In a separating equilibrium, the employer pays the worker a wage equal to her true ability, and the worker chooses the education level that maximizes her payoff, given the wage rate.

To find the conditions for a separating equilibrium, we need to consider the incentive compatibility constraints and the participation constraints of the workers. The incentive compatibility constraints ensure that the workers do not have an incentive to deviate from their equilibrium strategies and mimic the other type. The participation constraints ensure that the workers do not have an incentive to opt out of the signaling game and choose zero education.

The incentive compatibility constraints are:

- For the H-type: $w_H - e_H/H \geq w_L - e_L/H$ - For the L-type: $w_L - e_L/L \geq w_H - e_H/L$

The participation constraints are:

- For the H-type: $w_H - e_H/H \geq H$ - For the L-type: $w_L - e_L/L \geq L$

Using the fact that $w_H = H$ and $w_L = L$ in a separating equilibrium, we can simplify the constraints as follows:

- For the H-type: $e_H \leq e_L$ - For the L-type: $e_L \leq e_H(L/H)$

The first constraint implies that the H-type chooses the lowest possible education level, which is equal to the education level of the L-type. The second constraint implies that the L-type chooses the highest possible education level, which is a fraction of the education level of the H-type. Therefore, a separating equilibrium exists if and only if:

$$e_H = e_L$$

$$e_L = e_H(L/H)$$

Solving for $e_H$ and $e_L$, we obtain:

$$e_H = \frac{H}{H-L}e_L$$

$$e_L = \frac{L}{H-L}e_H$$

Substituting $e_L$ into $e_H$, we get:

$$e_H = \frac{H}{H-L}\frac{L}{H-L}e_H$$

Simplifying, we get:

$$e_H = \sqrt{\frac{HL}{(H-L)^2}}$$

$$e_L = \sqrt{\frac{HL}{(H-L)^2}\frac{L}{H}}$$

These are the education levels that satisfy the incentive compatibility and the participation constraints of the workers in a separating equilibrium. The corresponding wage rates are:

$$w_H = H$$

$$w_L = L$$

The payoff of the H-type in a separating equilibrium is:

$$\pi_H = H - \sqrt{\frac{HL}{(H-L)^2}}$$

The payoff of the L-type in a separating equilibrium is:

$$\pi_L = L - \sqrt{\frac{HL}{(H-L)^2}\frac{L}{H}}$$

The social welfare in a separating equilibrium is:

$$W = p\pi_H + (1-p)\pi_L$$

A separating equilibrium is efficient if it maximizes the social welfare, and equitable if it maximizes the expected utility of the workers. We will compare the efficiency and the equity of the separating equilibrium with the other equilibria in the next section.

## 3.2 Pooling Equilibrium

A pooling equilibrium is a strategy profile where both types of workers choose the same education level, and the employer cannot distinguish between them. In other words, a pooling equilibrium is a pair of education levels, $(e_H, e_L)$, such that $e_H = e_L = e^*$, and a pair of wage rates, $(w_H, w_L)$, such that $w_H = w_L = w^*$. In a pooling equilibrium, the employer pays the worker a wage equal to the average ability of the population, and the worker chooses the education level that maximizes her payoff, given the wage rate.

To find the conditions for a pooling equilibrium, we need to consider the incentive compatibility constraints and the participation constraints of the workers. The incentive compatibility constraints ensure that the workers do not have an incentive to deviate from their equilibrium strategies and separate themselves from the other type. The participation constraints ensure that the workers do not have an incentive to opt out of the signaling game and choose zero education.

The incentive compatibility constraints are:

- For the H-type: $w^* - e^*/H \geq H - e/H$ - For the L-type: $w^* - e^*/L \geq L - e/L$

The participation constraints are:

- For the H-type: $w^* - e^*/H \geq H$ - For the L-type: $w^* - e^*/L \geq L$

Using the fact that $w^* = pH + (1-p)L$ in a pooling equilibrium, we can simplify the constraints as follows:

- For the H-type: $e^* \leq e$ - For the L-type: $e^* \geq e(L/H)$

The first constraint implies that the H-type chooses the lowest possible education level, which is equal to the education level of the L-type. The second constraint implies that the L-type chooses the highest possible education level, which is a fraction of the education level of the H-type. Therefore, a pooling equilibrium exists if and only if:

$$e^* = e$$

$$e^* = e(L/H)$$

Solving for $e^*$ and $e$, we obtain:

$$e^* = \frac{H}{H-L}e$$

$$e = \frac{L}{H-L}e^*$$

Substituting $e$ into $e^*$, we get:

$$e^* = \frac{H}{H-L}\frac{L}{H-L}e^*$$

Simplifying, we get:

$$e^* = \sqrt{\frac{HL}{(H-L)^2}}$$

This is the education level that satisfies the incentive compatibility and the participation constraints of the workers in a pooling equilibrium. The corresponding wage rate is:

$$w^* = pH + (1-p)L$$

The payoff of the H-type in a pooling equilibrium is:

$$\pi_H = pH + (1-p)L - \sqrt{\frac{HL}{(H-L)^2}}$$

The payoff of the L-type in a pooling equilibrium is:

$$\pi_L = pH + (1-p)L - \sqrt{\frac{HL}{(H-L)^2}}$$

The social welfare in a pooling equilibrium is:

$$W = p\pi_H + (1-p)\pi_L$$

A pooling equilibrium is efficient if it maximizes the social welfare, and equitable if it maximizes the expected utility of the workers. We will compare the efficiency and the equity of the pooling equilibrium with the other equilibria in the next section.

## 3.3 Comparison of Equilibria

In this section, we compare the efficiency and the equity of the separating equilibrium and the pooling equilibrium. We also discuss the existence and the uniqueness of the equilibria, and the conditions under which one equilibrium dominates the other.

We first compare the efficiency of the equilibria, which is measured by the social welfare, $W$. The social welfare is the sum of the expected utilities of the workers, weighted by their proportions in the population. The higher the social welfare, the more efficient the equilibrium.

The social welfare in the separating equilibrium is:

$$W_S = p(H - \sqrt{\frac{HL}{(H-L)^2}}) + (1-p)(L - \sqrt{\frac{HL}{(H-L)^2}}\frac{L}{H})$$

The social welfare in the pooling equilibrium is:

$$W_P = p(pH + (1-p)L - \sqrt{\frac{HL}{(H-L)^2}}) + (1-p)(pH + (1-p)L - \sqrt{\frac{HL}{(H-L)^2}})$$

Simplifying, we get:

$$W_S = pH + (1-p)L - \sqrt{\frac{HL}{(H-L)^2}}$$

$$W_P = pH + (1-p)L - 2\sqrt{\frac{HL}{(H-L)^2}}$$

Comparing the two expressions, we can see that:

$$W_S > W_P$$

This means that the separating equilibrium is more efficient than the pooling equilibrium, as it generates a higher social welfare. The intuition is that the separating equilibrium allocates the workers to the firms according to their true abilities, while the pooling equilibrium allocates them randomly. The separating equilibrium eliminates the information asymmetry and the adverse selection problem, while the pooling equilibrium preserves them. The separating equilibrium also reduces the wasteful signaling expenditure, as the workers choose the lowest possible education level that separates them from the other type.

We next compare the equity of the equilibria, which is measured by the expected utility of the workers, $\pi$. The expected utility of the workers is the difference between their expected wage and their cost of education. The higher the expected utility, the more equitable the equilibrium.

The expected utility of the H-type in the separating equilibrium is:

$$\pi_{H,S} = H - \sqrt{\frac{HL}{(H-L)^2}}$$

The expected utility of the L-type in the separating equilibrium is:

$$\pi_{L,S} = L - \sqrt{\frac{HL}{(H-L)^2}\frac{L}{H}}$$

The expected utility of the H-type in the pooling equilibrium is:

$$\pi_{H,P} = pH + (1-p)L - \sqrt{\frac{HL}{(H-L)^2}}$$

The expected utility of the L-type in the pooling equilibrium is:

$$\pi_{L,P} = pH + (1-p)L - \sqrt{\frac{HL}{(H-L)^2}}$$

Comparing the two expressions, we can see that:

$$\pi_{H,S} > \pi_{H,P}$$

$$\pi_{L,S} < \pi_{L,P}$$

This means that the separating equilibrium is more equitable for the H-types, but less equitable for the L-types, than the pooling equilibrium. The intuition is that the separating equilibrium rewards the H-types for their higher ability and productivity, while the pooling equilibrium penalizes them for their lower signaling expenditure. The separating equilibrium also punishes the L-types for their lower ability and productivity, while the pooling equilibrium subsidizes them for their higher signaling expenditure. The separating equilibrium creates a gap between the H-types and the L-types, while the pooling equilibrium closes the gap.

We finally discuss the existence and the uniqueness of the equilibria, and the conditions under which one equilibrium dominates the other. The existence of the equilibria depends on the parameters of the model, such as the proportion of H-types, $p$, the abilities of the workers, $H$ and $L$, and the cost function of education, $e/\theta$. The uniqueness of the equilibria depends on the stability of the equilibria, which is determined by the best responses of the workers and the employers. The dominance of the equilibria depends on the preferences of the workers and the employers, and the trade-off between efficiency and equity.

The separating equilibrium exists if and only if the education levels of the workers are positive and finite, and satisfy the incentive compatibility and the participation constraints. This requires that the abilities of the workers are sufficiently different, and that the cost function of education is sufficiently convex. The separating equilibrium is unique and stable if and only if the best responses of the workers and the employers are monotonic and consistent. This requires that the education level of the worker is a decreasing function of the wage rate, and that the wage rate of the employer is an increasing function of the education level.

The pooling equilibrium exists if and only if the education level of the workers is positive and finite, and satisfies the incentive compatibility and the participation constraints. This requires that the abilities of the workers are sufficiently similar, and that the cost function of education is sufficiently concave. The pooling equilibrium is unique and stable if and only if the best responses of the workers and the employers are monotonic and consistent. This requires that the education level of the worker is a decreasing function of the wage rate, and that the wage rate of the employer is a constant function of the education level.

The separating equilibrium dominates the pooling equilibrium if and only if the workers and the employers prefer efficiency over equity, and are willing to accept the inequality and the signaling cost that the separating equilibrium entails. The pooling equilibrium dominates the separating equilibrium if and only if the workers and the employers prefer equity over efficiency, and are willing to accept the information asymmetry and the adverse selection that the pooling equilibrium entails.

# 4 The Spence Model with Grade Inflation

In this section, we extend the Spence model of education to account for grade inflation. We assume that the employer is either informed or uninformed about the grade inflation, and that the grade inflation affects either the cost or the signal of education. We analyze how grade inflation affects the equilibria, the efficiency, and the welfare of the signaling game.

We consider a population of workers, who are either of high-ability ($H$) or low-ability ($L$), where $H > L > 0$. The ability of a worker is her expected productivity in the firm, and is known to the worker but not to the employer. The proportion of H-types in the population is $p$, where $0 < p < 1$. A worker can choose to acquire $e$ units of education, where $e$ is any nonnegative number. The cost of education for a worker of type $\theta$ is $e/\theta$, where $\theta \in \{H, L\}$. This implies that H-types have a lower marginal cost of education than L-types.

The employer observes the education level of a worker, but not her ability. The employer forms an estimate of the worker's ability based on the conditional expectation of $\theta$ given $e$, denoted by $IE(\theta/e)$. The employer pays the worker a wage equal to this estimate, which is the market clearing wage under perfect competition. The payoff of a worker of type $\theta$ who chooses $e$ units of education is $IE(\theta/e) - e/\theta$. The payoff of the employer is zero, as he pays the worker exactly her expected productivity."

The signaling game consists of the same stages as in the basic Spence model, except that we introduce grade inflation as a phenomenon that affects either the cost or the signal of education. Grade inflation is a phenomenon where the average grades of students increase over time, without a corresponding increase in their academic performance or ability[1]. Grade inflation can lead to a compression of grades toward the top of the scale, which may affect the signaling value of education[23].

We assume that grade inflation occurs at a rate of g, where 0 ¡ g ¡ 1. We consider two scenarios: one where the employer is informed about the grade inflation and adjusts his beliefs accordingly, and another where the employer is uninformed about the grade inflation and bases his beliefs on the historical distribution of grades. We also consider two cases: one where grade inflation affects the cost of education, and another where grade inflation affects the signal of education.

## 4.1 Case 1: Grade Inflation Affects the Cost of Education

In this case, we assume that grade inflation reduces the cost of education for both types of workers, as they can obtain higher grades with less effort. We assume that the cost function of education for a worker of type $\theta$ is $e/(\theta + g)$, where $g$ is the grade inflation rate. This implies that the marginal cost of education decreases as $g$ increases.

We follow the same steps as in the basic Spence model to derive the conditions for a separating equilibrium and a pooling equilibrium, but we use the modified cost function of education. We then compare the efficiency and the welfare of the equilibria, and discuss their existence and uniqueness.

## 4.2 Separating Equilibrium

A separating equilibrium is a strategy profile where the H-types and the L-types choose different education levels, and the employer can perfectly infer the type of the worker from the education level. In other words, a separating equilibrium is a pair of education levels, $(e_H, e_L)$, such that $e_H > e_L$, and a pair of wage rates, $(w_H, w_L)$, such that $w_H = H$ and $w_L = L$. In a separating equilibrium, the employer pays the worker a wage equal to her true ability, and the worker chooses the education level that maximizes her payoff, given the wage rate.

To find the conditions for a separating equilibrium, we need to consider the incentive compatibility constraints and the participation constraints of the workers. The incentive compatibility constraints ensure that the workers do not have an incentive to deviate from their equilibrium strategies and mimic the other type. The participation constraints ensure that the workers do not have an incentive to opt out of the signaling game and choose zero education.

The incentive compatibility constraints are:

- For the H-type: $w_H - e_H/(H+g) \geq w_L - e_L/(H+g)$ - For the L-type: $w_L - e_L/(L+g) \geq w_H - e_H/(L+g)$

The participation constraints are:

- For the H-type: $w_H - e_H/(H+g) \geq H$ - For the L-type: $w_L - e_L/(L+g) \geq L$

Using the fact that $w_H = H$ and $w_L = L$ in a separating equilibrium, we can simplify the constraints as follows:

- For the H-type: $e_H \leq e_L$ - For the L-type: $e_L \leq e_H \frac{L+g}{H+g}$

The first constraint implies that the H-type chooses the lowest possible education level, which is equal to the education level of the L-type. The second constraint implies that the L-type chooses the highest possible education level, which is a fraction of the education level of the H-type. Therefore, a separating equilibrium exists if and only if:

$$e_H = e_L$$

$$e_L = e_H \frac{L+g}{H+g}$$

Solving for $e_H$ and $e_L$, we obtain:

$$e_H = \frac{H+g}{H-L+2g} e_L$$

$$e_L = \frac{L+g}{H-L+2g} e_H$$

Substituting $e_L$ into $e_H$, we get:

$$e_H = \frac{H+g}{H-L+2g} \frac{L+g}{H-L+2g} e_H$$

Simplifying, we get:

$$e_H = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

$$e_L = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}} \frac{L+g}{H+g}$$

These are the education levels that satisfy the incentive compatibility and the participation constraints of the workers in a separating equilibrium. The corresponding wage rates are:

$$w_H = H$$

$$w_L = L$$

The payoff of the H-type in a separating equilibrium is:

$$\pi_H = H - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The payoff of the L-type in a separating equilibrium is:

$$\pi_L = L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}\frac{L+g}{H+g}$$

The social welfare in a separating equilibrium is:

$$W = p\pi_H + (1-p)\pi_L$$

A separating equilibrium is efficient if it maximizes the social welfare, and equitable if it maximizes the expected utility of the workers. We will compare the efficiency and the equity of the separating equilibrium with the other equilibria in the next section.

## 4.3 Pooling Equilibrium

A pooling equilibrium is a strategy profile where both types of workers choose the same education level, and the employer cannot distinguish between them. In other words, a pooling equilibrium is a pair of education levels, $(e_H, e_L)$, such that $e_H = e_L = e^*$, and a pair of wage rates, $(w_H, w_L)$, such that $w_H = w_L = w^*$. In a pooling equilibrium, the employer pays the worker a wage equal to the average ability of the population, and the worker chooses the education level that maximizes her payoff, given the wage rate.

To find the conditions for a pooling equilibrium, we need to consider the incentive compatibility constraints and the participation constraints of the workers. The incentive compatibility constraints ensure that the workers do not have an incentive to deviate from their equilibrium strategies and separate themselves from the other type. The participation constraints ensure that the workers do not have an incentive to opt out of the signaling game and choose zero education.

The incentive compatibility constraints are:

- For the H-type: $w^* - e^*/(H+g) \geq H - e/(H+g)$ - For the L-type: $w^* - e^*/(L+g) \geq L - e/(L+g)$

The participation constraints are:

- For the H-type: $w^* - e^*/(H+g) \geq H$ - For the L-type: $w^* - e^*/(L+g) \geq L$

Using the fact that $w^* = pH + (1-p)L$ in a pooling equilibrium, we can simplify the constraints as follows:

- For the H-type: $e^* \leq e$ - For the L-type: $e^* \geq e(L+g)/(H+g)$

The first constraint implies that the H-type chooses the lowest possible education level, which is equal to the education level of the L-type. The second constraint implies that the L-type chooses the highest possible education level, which is a fraction of the education level of the H-type. Therefore, a pooling equilibrium exists if and only if:

$$e^* = e$$

$$e^* = e(L+g)/(H+g)$$

Solving for $e^*$ and $e$, we obtain:

$$e^* = \frac{H+g}{H-L+2g}e$$

$$e = \frac{L+g}{H-L+2g}e^*$$

Substituting $e$ into $e^*$, we get:

$$e^* = \frac{H+g}{H-L+2g}\frac{L+g}{H-L+2g}e^*$$

Simplifying, we get:

$$e^* = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

This is the education level that satisfies the incentive compatibility and the participation constraints of the workers in a pooling equilibrium. The corresponding wage rate is:

$$w^* = pH + (1-p)L$$

The payoff of the H-type in a pooling equilibrium is:

$$\pi_H = pH + (1-p)L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The payoff of the L-type in a pooling equilibrium is:

$$\pi_L = pH + (1-p)L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The social welfare in a pooling equilibrium is:

$$W = p\pi_H + (1-p)\pi_L$$

A pooling equilibrium is efficient if it maximizes the social welfare, and equitable if it maximizes the expected utility of the workers. We will compare the efficiency and the equity of the pooling equilibrium with the other equilibria in the next section.

## 4.4 Comparison of Equilibria

In this section, we compare the efficiency and the equity of the separating equilibrium and the pooling equilibrium under grade inflation. We also discuss the effect of the employer's information and the grade inflation rate on the equilibria, the efficiency, and the welfare of the signaling game.

We first compare the efficiency of the equilibria, which is measured by the social welfare, $W$. The social welfare is the sum of the expected utilities of the workers, weighted by their proportions in the population. The higher the social welfare, the more efficient the equilibrium.

The social welfare in the separating equilibrium is:

$$W_S = p(H - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}) + (1-p)(L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}\frac{L+g}{H+g})$$

The social welfare in the pooling equilibrium is:

$$W_P = p(pH + (1-p)L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}) + (1-p)(pH + (1-p)L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}})$$

Simplifying, we get:

$$W_S = pH + (1-p)L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

$$W_P = pH + (1-p)L - 2\sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

Comparing the two expressions, we can see that:

$$W_S > W_P$$

This means that the separating equilibrium is more efficient than the pooling equilibrium, as it generates a higher social welfare. The intuition is the same as in the basic Spence model, but the difference in social welfare is smaller due to the grade inflation. Grade inflation reduces the signaling value of education and makes it harder for the workers to separate themselves from the other type. Grade inflation also reduces the cost of education and makes it easier for the workers to acquire the same level of education as the other type. Therefore, grade inflation reduces the efficiency gain from the separating equilibrium and increases the efficiency loss from the pooling equilibrium.

We next compare the equity of the equilibria, which is measured by the expected utility of the workers, $\pi$. The expected utility of the workers is the difference between their expected wage and their cost of education. The higher the expected utility, the more equitable the equilibrium.

The expected utility of the H-type in the separating equilibrium is:

$$\pi_{H,S} = H - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The expected utility of the L-type in the separating equilibrium is:

$$\pi_{L,S} = L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}\frac{L+g}{H+g}}$$

The expected utility of the H-type in the pooling equilibrium is:

$$\pi_{H,P} = pH + (1-p)L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The expected utility of the L-type in the pooling equilibrium is:

$$\pi_{L,P} = pH + (1-p)L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

Comparing the two expressions, we can see that:

$$\pi_{H,S} > \pi_{H,P}$$

$$\pi_{L,S} < \pi_{L,P}$$

This means that the separating equilibrium is more equitable for the H-types, but less equitable for the L-types, than the pooling equilibrium. The intuition is the same as in the basic Spence model, but the difference in expected utility is smaller due to the grade inflation. Grade inflation reduces the signaling value of education and makes it harder for the H-types to reveal their higher ability and productivity. Grade inflation also reduces the cost of education and makes it easier for the L-types to conceal their lower ability and productivity. Therefore, grade inflation reduces the equity gain for the H-types and the equity loss for the L-types from the separating equilibrium.

We finally discuss the effect of the employer's information and the grade inflation rate on the equilibria, the efficiency, and the welfare of the signaling game. The employer's information is captured by the parameter $g$, which represents the grade inflation rate that the employer is aware of. The grade inflation rate is captured by the parameter $g$, which represents the actual grade inflation rate that affects the cost or the signal of education. We consider two scenarios: one where the employer is informed about the grade inflation and adjusts his beliefs accordingly, and another where the employer is uninformed about the grade inflation and bases his beliefs on the historical distribution of grades.

## 4.5 Scenario 1: Informed Employer

In this scenario, we assume that the employer is informed about the grade inflation and adjusts his beliefs accordingly. This means that the employer knows the actual grade inflation rate, $g$, and uses it to form his estimate of the worker's ability, $IE(\theta/e)$. In other words, the employer's information is equal to the grade inflation rate, $g = g$.

In this scenario, the separating equilibrium and the pooling equilibrium are the same as in the previous section, as the grade inflation affects only the cost or the signal of education, but not the employer's beliefs. Therefore, the efficiency and the equity of the equilibria are also the same as in the previous section.

However, the existence and the uniqueness of the equilibria depend on the value of $g$. As $g$ increases, the cost of education decreases, and the signal of education becomes less informative. This makes it easier for the workers to pool and harder for them to separate. Therefore, the separating equilibrium becomes less likely and the pooling equilibrium becomes more likely as $g$ increases.

To see this, we can compare the education levels of the workers in the separating equilibrium and the pooling equilibrium, as functions of $g$. The education level of the H-type in the separating equilibrium is:

$$e_{H,S} = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The education level of the L-type in the separating equilibrium is:

$$e_{L,S} = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}\frac{L+g}{H+g}}$$

The education level of the workers in the pooling equilibrium is:

$$e^* = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

We can see that $e_{H,S}$ and $e^*$ are increasing functions of $g$, while $e_{L,S}$ is a decreasing function of $g$. This means that as $g$ increases, the H-type chooses a higher education level, the L-type chooses a lower education level, and both types converge to the same education level. Therefore, the separating equilibrium disappears and the pooling equilibrium emerges as $g$ increases.

We can also see that $e_{H,S}$ and $e_{L,S}$ are equal when $g = 0$, and $e_{H,S}$ and $e^*$ are equal when $g = H - L$. This means that when $g = 0$, there is a unique separating equilibrium, and when $g = H - L$, there is a unique pooling equilibrium. When $0 < g < H - L$, there are multiple equilibria, and when $g > H - L$, there are no equilibria.

### 4.6 Scenario 2: Uninformed Employer

In this scenario, we assume that the employer is uninformed about the grade inflation and bases his beliefs on the historical distribution of grades. This means that the employer does not know the actual grade inflation rate, $g$, and uses the historical grade inflation rate, $g$, to form his estimate of the worker's ability, $IE(\theta/e)$. In other words, the employer's information is different from the grade inflation rate, $g \neq g$.

In this scenario, the separating equilibrium and the pooling equilibrium are different from the previous section, as the grade inflation affects not only the cost or the signal of education, but also the employer's beliefs. Therefore, the efficiency and the equity of the equilibria are also different from the previous section.

However, the existence and the uniqueness of the equilibria still depend on the value of $g$. As $g$ increases, the cost of education decreases, and the signal of education becomes less informative. This makes it easier for the workers to pool and harder for them to separate. Therefore, the separating equilibrium becomes less likely and the pooling equilibrium becomes more likely as $g$ increases.

To see this, we can compare the education levels of the workers in the separating equilibrium and the pooling equilibrium, as functions of $g$. The education level of the H-type in the separating equilibrium is:

$$e_{H,S} = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The education level of the L-type in the separating equilibrium is:

$$e_{L,S} = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}\frac{L+g}{H+g}$$

The education level of the workers in the pooling equilibrium is:

$$e^* = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

We can see that $e_{H,S}$ and $e^*$ are increasing functions of $g$, while $e_{L,S}$ is a decreasing function of $g$. This means that as $g$ increases, the H-type chooses a higher education level, the L-type chooses a lower education level, and both types converge to the same education level. Therefore, the separating equilibrium disappears and the pooling equilibrium emerges as $g$ increases.

We can also see that $e_{H,S}$ and $e_{L,S}$ are equal when $g = 0$, and $e_{H,S}$ and $e^*$ are equal when $g = H - L$. This means that when $g = 0$, there is a unique separating equilibrium, and when $g = H - L$, there is a unique pooling equilibrium. When $0 < g < H - L$, there are multiple equilibria, and when $g > H - L$, there are no equilibria.

The difference between this scenario and the previous one is that the employer's beliefs are affected by the grade inflation. This means that the employer does not pay the worker a wage equal to her true ability or the average ability of the population, but a wage equal to his estimate of the worker's ability, which is based on the historical grade inflation rate, $g$. In other words, the employer's beliefs are biased by the grade inflation.

The employer's beliefs are captured by the conditional expectation of $\theta$ given $e$, denoted by IE($\theta/e$). The employer uses the historical grade inflation rate, $g$, to form his estimate of the worker's ability, IE($\theta/e$). The employer pays the worker a wage equal to this estimate, which is the market clearing wage under perfect competition.

The employer's beliefs are given by the following formula:

$$IE(\theta/e) = p\frac{f_H(e)}{f_H(e) + f_L(e)}H + (1-p)\frac{f_L(e)}{f_H(e) + f_L(e)}L$$

where $f_H(e)$ and $f_L(e)$ are the probability density functions of the education levels of the H-types and the L-types, respectively, under the historical grade inflation rate, $g$. These functions are given by:

$$f_H(e) = \frac{H - L + 2g}{(H+g)(L+g)}e^{-\frac{(H-L+2g)^2}{(H+g)(L+g)}e^2}$$

$$f_L(e) = \frac{H - L + 2g}{(H+g)(L+g)}\frac{L+g}{H+g}e^{-\frac{(H-L+2g)^2}{(H+g)(L+g)}e^2}$$

These functions are derived from the assumption that the education levels of the workers follow a normal distribution with mean zero and variance $\frac{(H+g)(L+g)}{(H-L+2g)^2}$, and that the L-types have a lower mean than the H-types by a factor of $\frac{L+g}{H+g}$.

Using these functions, we can calculate the employer's beliefs for any given education level, $e$, and historical grade inflation rate, $g$. We can then compare the employer's beliefs with the true abilities of the workers, $H$ and $L$, and the average ability of the population, $pH + (1-p)L$. This will allow us to see how the grade inflation affects the employer's beliefs and the wages of the workers.

We can plot the employer's beliefs as a function of the education level, $e$, and the historical grade inflation rate, $g$, for different values of the actual grade inflation rate, $g$. Figure 1 shows an example of such a plot, where we assume that $H = 2$, $L = 1$, and $p = 0.5$.

[Figure 1]

Figure 1: Employer's beliefs as a function of education level and historical grade inflation rate

We can see from Figure 1 that the employer's beliefs are increasing in the education level, $e$, and decreasing in the historical grade inflation rate, $g$. This means that the employer assigns a higher probability to the worker being a H-type when the worker has a higher education level or when the employer observes a lower grade inflation rate. We can also see that the employer's beliefs are affected by the actual grade inflation rate, $g$. When $g > g$, the employer underestimates the worker's ability, and when $g < g$, the employer overestimates the worker's ability. This means that the employer's beliefs are biased by the grade inflation.

The bias in the employer's beliefs has implications for the wages of the workers. The employer pays the worker a wage equal to his estimate of the worker's ability, IE$(\theta/e)$. Therefore, the wage of the worker is equal to the employer's beliefs, multiplied by the expected ability of the H-type, $H$, plus the complement of the employer's beliefs, multiplied by the expected ability of the L-type, $L$. In other words, the wage of the worker is given by:

$$w = IE(\theta/e)H + (1 - IE(\theta/e))L$$

Using this formula, we can calculate the wage of the worker for any given education level, $e$, historical grade inflation rate, $g$, and actual grade inflation rate, $g$. We can then compare the wage of the worker with the true abilities of the workers, $H$ and $L$, and the average ability of the population, $pH + (1 - p)L$. This will allow us to see how the grade inflation affects the wages of the workers. We can plot the wage of the worker as a function of the education level, $e$, and the historical grade inflation rate, $g$, for different values of the actual grade inflation rate, $g$. Figure 2 shows an example of such a plot, where we assume that $H = 2$, $L = 1$, and $p = 0.5$.

[Figure 2]

Figure 2: Wage of the worker as a function of education level and historical grade inflation rate

We can see from Figure 2 that the wage of the worker is increasing in the education level, $e$, and decreasing in the historical grade inflation rate, $g$. This means that the worker receives a higher wage when she has a higher education level or when the employer observes a lower grade inflation rate. We can also see that the wage of the worker is affected by the actual grade inflation rate, $g$. When $g > g$, the worker receives a lower wage than her true ability, and when $g < g$, the worker receives a higher wage than her true ability. This means that the wage of the worker is biased by the grade inflation.

The bias in the wage of the worker has implications for the efficiency and the equity of the equilibria. The efficiency of the equilibria is measured by the social welfare, $W$, which is the sum of the expected utilities of the workers, weighted by their proportions in the population. The equity of the equilibria is measured by the expected utility of the workers, $\pi$, which is the difference between their expected wage and their cost of education. The higher the social welfare and the expected utility, the more efficient and equitable the equilibrium.

We can calculate the social welfare and the expected utility of the workers in the separating equilibrium and the pooling equilibrium, using the wage function and the cost function of education. We can then compare the efficiency and the equity of the equilibria, and discuss how they are affected by the grade inflation. We can also plot the social welfare and the expected utility of the workers as functions of the education level, $e$, and the historical grade inflation rate, $g$, for different values of the actual grade inflation rate, $g$. Figure 3 shows an example of such a plot, where we assume that $H = 2$, $L = 1$, and $p = 0.5$.

[Figure 3]

Figure 3: Social welfare and expected utility of the workers as functions of education level and historical grade inflation rate

We can see from Figure 3 that the social welfare and the expected utility of the workers are decreasing in the historical grade inflation rate, $g$. This means that the efficiency and the equity of the equilibria are lower when the employer observes a higher grade inflation rate. We can also see that the social welfare and the expected utility of the workers are affected by the actual grade inflation rate, $g$. When $g > g$, the social welfare and the expected utility of the workers are lower than when $g = g$, and when $g < g$, the social welfare and the expected utility of the workers are higher than when $g = g$. This means that the efficiency and the equity of the equilibria are biased by the grade inflation.

The bias in the efficiency and the equity of the equilibria has implications for the existence and the uniqueness of the equilibria. The existence of the equilibria depends on the parameters of the model, such as the proportion of H-types, $p$, the abilities of the workers, $H$ and $L$, the cost function of education, $e/(\theta+g)$, and the employer's beliefs, $\mathrm{IE}(\theta/e)$. The uniqueness of the equilibria depends on the stability of the equilibria, which is determined by the best responses of the workers and the employers. The dominance of the equilibria depends on the preferences of the workers and the employers, and the trade-off between efficiency and equity.

We can use the same logic as in the previous section to analyze the existence and the uniqueness of the equilibria, but we need to take into account the effect of the grade inflation on the employer's beliefs and the wages of the workers. As $g$ increases, the cost of education decreases, the signal of education becomes less informative, and the employer's beliefs become more biased. This makes it easier for the workers to pool and harder for them to separate. Therefore, the separating equilibrium becomes less likely and the pooling equilibrium becomes more likely as $g$ increases.

To see this, we can compare the education levels of the workers in the separating equilibrium and the pooling equilibrium, as functions of $g$. The education level of the H-type in the separating equilibrium is:

$$e_{H,S} = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The education level of the L-type in the separating equilibrium is:

$$e_{L,S} = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}\frac{L+g}{H+g}$$

The education level of the workers in the pooling equilibrium is:

$$e^* = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

We can see that $e_{H,S}$ and $e^*$ are increasing functions of $g$, while $e_{L,S}$ is a decreasing function of $g$. This means that as $g$ increases, the H-type chooses a higher education level, the L-type chooses a lower education level, and both types converge to the same education level. Therefore, the separating equilibrium disappears and the pooling equilibrium emerges as $g$ increases.

We can also see that $e_{H,S}$ and $e_{L,S}$ are equal when $g = 0$, and $e_{H,S}$ and $e^*$ are equal when $g = H - L$. This means that when $g = 0$, there is a unique separating equilibrium, and when $g = H - L$, there is a unique pooling equilibrium. When $0 < g < H - L$, there are multiple equilibria, and when $g > H - L$, there are no equilibria.

The dominance of the equilibria depends on the preferences of the workers and the employers, and the trade-off between efficiency and equity. The separating equilibrium dominates the pooling equilibrium if and only if the workers and the employers prefer efficiency over equity, and are willing to accept the inequality and the signaling cost that the separating equilibrium entails. The pooling equilibrium dominates the separating equilibrium if and only if the workers and the employers prefer equity over efficiency, and are willing to accept the information asymmetry and the adverse selection that the pooling equilibrium entails.

However, the grade inflation affects the efficiency and the equity of the equilibria, and therefore the dominance of the equilibria. As $g$ increases, the efficiency and the equity of the equilibria decrease, and the difference between the separating equilibrium and the pooling equilibrium becomes smaller. Therefore, the grade inflation reduces the preference for the separating equilibrium and increases the preference for the pooling equilibrium. When $g$ is sufficiently high, the pooling equilibrium may dominate the separating equilibrium, even if the workers and the employers prefer efficiency over equity.

This is because the grade inflation makes the separating equilibrium less efficient and less equitable, as it reduces the signaling value of education and creates a bias in the employer's beliefs and the wages of the workers. Therefore, the grade inflation reduces the preference for the separating equilibrium and increases the preference for the pooling equilibrium. When $g$ is sufficiently high, the pooling equilibrium may dominate the separating equilibrium, even if the workers and the employers prefer efficiency over equity.

This concludes our analysis of the case where grade inflation affects the cost of education. In the next section, we will analyze the case where grade inflation affects the signal of education.

## 4.7 Case 2: Grade Inflation Affects the Signal of Education

In this case, we assume that grade inflation reduces the cost of education for both types of workers, as they can obtain higher grades with less effort. We assume that the cost function of education for a worker of type $\theta$ is $e/(\theta + g)$, where $g$ is the grade inflation rate. This implies that the marginal cost of education decreases as $g$ increases.

We follow the same steps as in the basic Spence model to derive the conditions for a separating equilibrium and a pooling equilibrium, but we use the modified signal function of education. We then compare the efficiency and the welfare of the equilibria, and discuss their existence and uniqueness.

## 4.8 Separating Equilibrium

A separating equilibrium is a strategy profile where the H-types and the L-types choose different education levels, and the employer can perfectly infer the type of the worker from the education level. In other words, a separating equilibrium is a pair of education levels, $(e_H, e_L)$, such that $e_H > e_L$, and a pair of wage rates, $(w_H, w_L)$, such that $w_H = H$ and $w_L = L$. In a separating equilibrium, the employer pays the worker a wage equal to her true ability, and the worker chooses the education level that maximizes her payoff, given the wage rate.

To find the conditions for a separating equilibrium, we need to consider the incentive compatibility constraints and the participation constraints of the workers. The incentive compatibility constraints ensure that the workers do not have an incentive to deviate from their equilibrium strategies and mimic the other type. The participation constraints ensure that the workers do not have an incentive to opt out of the signaling game and choose zero education.

The incentive compatibility constraints are:

- For the H-type: $w_H - e_H/H \geq w_L - e_L/H$ - For the L-type: $w_L - e_L/L \geq w_H - e_H/L$

The participation constraints are:

- For the H-type: $w_H - e_H/H \geq H$ - For the L-type: $w_L - e_L/L \geq L$

Using the fact that $w_H = H$ and $w_L = L$ in a separating equilibrium, we can simplify the constraints as follows:

- For the H-type: $e_H \leq e_L(H+g)/(L+g)$ - For the L-type: $e_L \leq e_H(L+g)/(H+g)$

The first constraint implies that the H-type chooses the lowest possible education level, which is a fraction of the education level of the L-type. The second constraint implies that the L-type chooses the highest possible education level, which is a fraction of the education level of the H-type. Therefore, a separating equilibrium exists if and only if:

$$e_H = e_L(H+g)/(L+g)$$

$$e_L = e_H(L+g)/(H+g)$$

Solving for $e_H$ and $e_L$, we obtain:

$$e_H = \frac{H+g}{H-L+2g} e_L$$

$$e_L = \frac{L+g}{H-L+2g} e_H$$

Substituting $e_L$ into $e_H$, we get:

$$e_H = \frac{H+g}{H-L+2g} \frac{L+g}{H-L+2g} e_H$$

Simplifying, we get:

$$e_H = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

$$e_L = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}} \frac{L+g}{H+g}$$

These are the education levels that satisfy the incentive compatibility and the participation constraints of the workers in a separating equilibrium. The corresponding wage rates are:

$$w_H = H$$

$$w_L = L$$

The payoff of the H-type in a separating equilibrium is:

$$\pi_H = H - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The payoff of the L-type in a separating equilibrium is:

$$\pi_L = L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}\frac{L+g}{H+g}}$$

The social welfare in a separating equilibrium is:

$$W = p\pi_H + (1-p)\pi_L$$

A separating equilibrium is efficient if it maximizes the social welfare, and equitable if it maximizes the expected utility of the workers. We will compare the efficiency and the equity of the separating equilibrium with the other equilibria in the next section.

## 4.9 Pooling Equilibrium

A pooling equilibrium is a strategy profile where both types of workers choose the same education level, and the employer cannot distinguish between them. In other words, a pooling equilibrium is a pair of education levels, $(e_H, e_L)$, such that $e_H = e_L = e^*$, and a pair of wage rates, $(w_H, w_L)$, such that $w_H = w_L = w^*$. In a pooling equilibrium, the employer pays the worker a wage equal to the average ability of the population, and the worker chooses the education level that maximizes her payoff, given the wage rate.

To find the conditions for a pooling equilibrium, we need to consider the incentive compatibility constraints and the participation constraints of the workers. The incentive compatibility constraints ensure that the workers do not have an incentive to deviate from their equilibrium strategies and separate themselves from the other type. The participation constraints ensure that the workers do not have an incentive to opt out of the signaling game and choose zero education.

The incentive compatibility constraints are:

- For the H-type: $w^* - e^*/H \geq H - e/(H+g)$ - For the L-type: $w^* - e^*/L \geq L - e/(L+g)$

The participation constraints are:

- For the H-type: $w^* - e^*/H \geq H$ - For the L-type: $w^* - e^*/L \geq L$

Using the fact that $w^* = pH + (1-p)L$ in a pooling equilibrium, we can simplify the constraints as follows:

- For the H-type: $e^* \leq e$ - For the L-type: $e^* \geq e(L+g)/(H+g)$

The first constraint implies that the H-type chooses the lowest possible education level, which is equal to the education level of the L-type. The second constraint implies that the L-type chooses the highest possible education level, which is a fraction of the education level of the H-type. Therefore, a pooling equilibrium exists if and only if:

$$e^* = e$$

$$e^* = e(L+g)/(H+g)$$

Solving for $e^*$ and $e$, we obtain:

$$e^* = \frac{H+g}{H-L+2g}e$$

$$e = \frac{L+g}{H-L+2g}e^*$$

Substituting $e$ into $e^*$, we get:

$$e^* = \frac{H+g}{H-L+2g}\frac{L+g}{H-L+2g}e^*$$

Simplifying, we get:

$$e^* = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

This is the education level that satisfies the incentive compatibility and the participation constraints of the workers in a pooling equilibrium. The corresponding wage rate is:

$$w^* = pH + (1-p)L$$

The payoff of the H-type in a pooling equilibrium is:

$$\pi_{H,P} = pH + (1-p)L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The payoff of the L-type in a pooling equilibrium is:

$$\pi_{L,P} = pH + (1-p)L - \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The social welfare in a pooling equilibrium is:

$$W_P = p\pi_{H,P} + (1-p)\pi_{L,P}$$

A pooling equilibrium is efficient if it maximizes the social welfare, and equitable if it maximizes the expected utility of the workers. We will compare the efficiency and the equity of the pooling equilibrium with the other equilibria in the next section.

## 4.10 Comparison of Equilibria

In the case where grade inflation affects the signal of education, the separating equilibrium and the pooling equilibrium have different efficiency and equity implications, depending on the employer's information and the grade inflation rate.

## 4.11 Scenario 1: Informed Employer

In Scenario 1, where the employer is informed about the grade inflation and adjusts his beliefs accordingly, the separating equilibrium is more efficient and more equitable for the H-types than the pooling equilibrium, as in the first case. However, the difference in efficiency and equity is smaller due to the grade inflation, as it reduces the signaling value of education and makes it harder for the H-types to reveal their higher ability and productivity. The existence and the uniqueness of the equilibria depend on the value of g, the actual grade inflation rate. As g increases, the separating equilibrium becomes less likely and the pooling equilibrium becomes more likely, as the cost of education decreases and the signal of education becomes less informative.

## 4.12 Scenario 2: Uninformed Employer

In Scenario 2, where the employer is uninformed about the grade inflation and bases his beliefs on the historical distribution of grades, the separating equilibrium and the pooling equilibrium have different wage rates, as the employer's beliefs are biased by the grade inflation. The employer pays the worker a wage equal to his estimate of the worker's ability, which is based on the historical grade inflation rate, g. The employer's beliefs are given by the following formula:

$$IE(\theta/e) = p\frac{f_H(e)}{f_H(e) + f_L(e)}H + (1-p)\frac{f_L(e)}{f_H(e) + f_L(e)}L$$

where $f_H(e)$ and $f_L(e)$ are the probability density functions of the education levels of the H-types and the L-types, respectively, under the historical grade inflation rate, g. These functions are given by:

$$f_H(e) = \frac{H - L + 2g}{(H+g)(L+g)}e^{-\frac{(H-L+2g)^2}{(H+g)(L+g)}e^2}$$

$$f_L(e) = \frac{H - L + 2g}{(H+g)(L+g)}\frac{L+g}{H+g}e^{-\frac{(H-L+2g)^2}{(H+g)(L+g)}e^2}$$

The effect of the grade inflation rate on the employer's beliefs and the wage rates in the separating and the pooling equilibria depends on the difference between g and g, the actual and the historical grade inflation rates. If g ¿ g, the employer underestimates the worker's ability and pays a lower wage than the true or the average ability. If g ¡ g, the employer overestimates the worker's ability and pays a higher wage than the true or the average ability. If g = g, the employer's beliefs are unbiased and the wage rates are the same as in Scenario 1.

Therefore, in Scenario 2, the separating equilibrium and the pooling equilibrium have different efficiency and equity implications, depending on the difference between g and g. If g ¿ g, the separating equilibrium is more efficient and more equitable for the H-types than the pooling equilibrium, as in Scenario 1. However, if g ¡ g, the separating equilibrium is less efficient and less equitable for the H-types than the pooling equilibrium, as the employer pays a higher wage to the L-types than their true ability. The existence and the uniqueness of the equilibria still depend on the value of g, the actual grade inflation rate, as in Scenario 1.

The education level of the L-type in the separating equilibrium is:

$$e_{L,S} = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}\frac{L+g}{H+g}}$$

We can see that $e_{L,S}$ is a decreasing function of $g$, while $e_{H,S}$ is an increasing function of $g$. This means that as $g$ increases, the L-type chooses a lower education level, the H-type chooses a higher education level, and both types diverge from the same education level. Therefore, the separating equilibrium emerges and the pooling equilibrium disappears as $g$ increases.

We can also see that $e_{L,S}$ and $e_{H,S}$ are equal when $g = 0$, and $e_{L,S}$ and $e^*$ are equal when $g = H - L$. This means that when $g = 0$, there is a unique pooling equilibrium, and when $g = H - L$, there is a unique separating equilibrium. When $0 < g < H - L$, there are multiple equilibria, and when $g > H - L$, there are no equilibria.

The difference between this scenario and the previous one is that the employer's beliefs are affected by the grade inflation. This means that the employer does not pay the worker a wage equal to her true ability or the average ability of the population, but a wage equal to his estimate of the worker's ability, which is based on the historical grade inflation rate, $g$. In other words, the employer's beliefs are biased by the grade inflation.

In Case 2, where grade inflation affects the signal of education, the employer's beliefs are also a function of the education level, $e$, and the historical grade inflation rate, $g$, for different values of the actual grade inflation rate, $g$. Figure 1B shows an example of such a plot, where we assume the same values of $H$, $L$, and $p$ as in Case 1.

[Figure 1B]

Figure 1B: Employer's beliefs as a function of education level and historical grade inflation rate

We can see from Figure 1B that the employer's beliefs are increasing in the education level, $e$, and decreasing in the historical grade inflation rate, $g$, as in Case 1. However, the slope of the employer's beliefs is steeper in Case 2 than in Case 1, as the signal of education is more informative in Case 2 than in Case 1. This means that the employer assigns a higher probability to the worker being a H-type when the worker has a higher education level or when the employer observes a lower grade inflation rate, and a lower probability when the worker has a lower education level or when the employer observes a higher grade inflation rate, in Case 2 than in Case 1.

We can also see that the employer's beliefs are affected by the actual grade inflation rate, $g$, as in Case 1. When $g > g$, the employer underestimates the worker's ability, and when $g < g$, the employer overestimates the worker's ability. This means that the employer's beliefs are biased by the grade inflation.

The bias in the employer's beliefs has implications for the wages of the workers, as in Case 1. The employer pays the worker a wage equal to his estimate of the worker's ability, IE($\theta/e$). Therefore, the wage of the worker is equal to the employer's beliefs, multiplied by the expected ability of the H-type, $H$, plus the complement of the employer's beliefs, multiplied by the expected ability of the L-type, $L$. In other words, the wage of the worker is given by:

$$w = IE(\theta/e)H + (1 - IE(\theta/e))L$$

This is a similar explanation/version of Figure 1 for Case 2, where grade inflation affects the signal of education.

The bias in the employer's beliefs has implications for the wages of the workers. The employer pays the worker a wage equal to his estimate of the worker's ability, IE($\theta/e$). Therefore, the wage of the worker is equal to the employer's beliefs, multiplied by the expected ability of the H-type, $H$, plus the complement of the employer's beliefs, multiplied by the expected ability of the L-type, $L$. In other words, the wage of the worker is given by:

$$w = IE(\theta/e)H + (1 - IE(\theta/e))L$$

Using this formula, we can calculate the wage of the worker for any given education level, $e$, historical grade inflation rate, $g$, and actual grade inflation rate, $g$. We can then compare the wage of the worker with the true abilities of the workers, $H$ and $L$, and the average ability of the population, $pH + (1 - p)L$. This will allow us to see how the grade inflation affects the wages of the workers. We can plot the wage of the worker as a function of the education level, $e$, and the historical grade inflation rate, $g$, for different values of the actual grade inflation rate, $g$. Figure 2B shows an example of such a plot, where we assume that $H = 2$, $L = 1$, and $p = 0.5$.

[Figure 2B]

Figure 2B: Wage of the worker as a function of education level and historical grade inflation rate

We can see from Figure 2B that the wage of the worker is increasing in the education level, $e$, and decreasing in the historical grade inflation rate, $g$. This means that the worker receives a higher wage when she has a higher education level or when the employer observes a lower grade inflation rate. We can also see that the wage of the worker is affected by the actual grade inflation rate, $g$. When $g > g$, the worker receives a lower wage than her true ability, and when $g < g$, the worker receives a higher wage than her true ability. This means that the wage of the worker is biased by the grade inflation.

The bias in the wage of the worker has implications for the efficiency and the equity of the equilibria. The efficiency of the equilibria is measured by the social welfare, $W$, which is the sum of the expected utilities of the workers, weighted by their proportions in the population. The equity of the equilibria is measured by the expected utility of the workers, $\pi$, which is the difference between their expected wage and their cost of education. The higher the social welfare and the expected utility, the more efficient and equitable the equilibrium.

We can calculate the social welfare and the expected utility of the workers in the separating equilibrium and the pooling equilibrium, using the wage function and the cost function of education. We can then compare the efficiency and the equity of the equilibria, and discuss how they are affected by the grade inflation. We can also plot the social welfare and the expected utility of the workers as functions of the education level, $e$, and the historical grade inflation rate, $g$, for different values of the actual grade inflation rate, $g$. Figure 3B shows an example of such a plot, where we assume that $H = 2$, $L = 1$, and $p = 0.5$.

[Figure 3B]

Figure 3B: Social welfare and expected utility of the workers as functions of education level and historical grade inflation rate

We can see from Figure 3B that the social welfare and the expected utility of the workers are decreasing in the historical grade inflation rate, $g$. This means that the efficiency and the equity of the equilibria are lower when the employer observes a higher grade inflation rate. We can also see that the social welfare and the expected utility of the workers are affected by the actual grade inflation rate, $g$. When $g > g$, the social welfare and the expected utility of the workers are lower than when $g = g$, and when $g < g$, the social welfare and the expected utility of the workers are higher than when $g = g$. This means that the efficiency and the equity of the equilibria are biased by the grade inflation.

The bias in the efficiency and the equity of the equilibria has implications for the existence and the uniqueness of the equilibria. The existence of the equilibria depends on the parameters of the model, such as the proportion of H-types, $p$, the abilities of the workers, $H$ and $L$, the cost function of education, $e/(\theta+g)$, and the employer's beliefs, $\mathrm{IE}(\theta/e)$. The uniqueness of the equilibria depends on the stability of the equilibria, which is determined by the best responses of the workers and the employers. The dominance of the equilibria depends on the preferences of the workers and the employers, and the trade-off between efficiency and equity.

We can use the same logic as in the previous section to analyze the existence and the uniqueness of the equilibria, but we need to take into account the effect of the grade inflation on the employer's beliefs and the wages of the workers. As $g$ increases, the signal of education increases, the cost of education increases, and the employer's beliefs become more biased. This makes it harder for the workers to pool and easier for them to separate. Therefore, the separating equilibrium becomes more likely and the pooling equilibrium becomes less likely as $g$ increases.

To see this, we can compare the education levels of the workers in the separating equilibrium and the pooling equilibrium, as functions of $g$. The education level of the H-type in the separating equilibrium is:

$$e_{H,S} = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

The education level of the L-type in the separating equilibrium is:

$$e_{L,S} = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}\frac{L+g}{H+g}}$$

The education level of the workers in the pooling equilibrium is:

$$e^* = \sqrt{\frac{(H+g)(L+g)}{(H-L+2g)^2}}$$

We can see that $e_{H,S}$ and $e^*$ are increasing functions of $g$, while $e_{L,S}$ is a decreasing function of $g$. This means that as $g$ increases, the H-type chooses a higher education level, the L-type chooses a lower education level, and both types diverge from the same education level. Therefore, the separating equilibrium emerges and the pooling equilibrium disappears as $g$ increases.

We can also see that $e_{H,S}$ and $e_{L,S}$ are equal when $g = 0$, and $e_{H,S}$ and $e^*$ are equal when $g = H-L$. This means that when $g = 0$, there is a unique pooling equilibrium, and when $g = H-L$, there is a unique separating equilibrium. When $0 < g < H - L$, there are multiple equilibria, and when $g > H - L$, there are no equilibria.

The dominance of the equilibria depends on the preferences of the workers and the employers, and the trade-off between efficiency and equity. The separating equilibrium dominates the pooling equilibrium if and only if the workers and the employers prefer efficiency over equity, and are willing to accept the inequality and the signaling cost that the separating equilibrium entails. The pooling equilibrium dominates the separating equilibrium if and only if the workers and the employers prefer equity over efficiency, and are willing to accept the information asymmetry and the adverse selection that the pooling equilibrium entails. However, the grade inflation affects the efficiency and the equity of the equilibria, and therefore the dominance of the equilibria.

# 5 Conclusion

In this paper, we have analyzed the Spence signaling model of education with grade inflation. We have considered two cases: one where grade inflation affects the cost of education, and another where grade inflation affects the signal of education. We have derived the conditions for a separating equilibrium and a pooling equilibrium in each case, and compared their efficiency and equity. We have also discussed the effect of the employer's information and the grade inflation rate on the equilibria, the efficiency, and the welfare of the signaling game.

We have found that grade inflation reduces the efficiency and the equity of the equilibria, as it reduces the signaling value of education and creates a bias in the employer's beliefs and the wages of the workers. We have also found that the separating equilibrium becomes more likely and the pooling equilibrium becomes less likely as the grade inflation rate increases, but the existence and the uniqueness of the equilibria depend on the value of the grade inflation rate and the employer's information.

Our analysis has some implications for the role of education in the labor market and the policy responses to grade inflation. Our analysis suggests that education is not only a source of human capital, but also a device to signal ability and productivity. Therefore, education may have positive externalities for the society, as it improves the allocation of workers to jobs and reduces the information asymmetry between workers and employers. However, our analysis also suggests that grade inflation may undermine the signaling role of education and distort the incentives of workers and employers. Therefore, grade inflation may have negative externalities for the society, as it reduces the efficiency and the equity of the labor market outcomes and creates a mismatch between workers and jobs.

Our analysis also suggests that the optimal policy response to grade inflation depends on the source and the extent of grade inflation, and the information structure of the labor market. If grade inflation is caused by the supply side of education, such as the leniency of teachers or the competition among schools, then the policy response may be to regulate the quality and the standards of education, and to monitor and disclose the grade inflation rates of different institutions. If grade inflation is caused by the demand side of education, such as the preferences of students or the expectations of employers, then the policy response may be to educate the students and the employers about the costs and benefits of education, and to encourage the use of alternative or complementary signals of ability and productivity, such as tests, portfolios, or references. In any case, the policy response should aim to restore the signaling value of education and to align the beliefs and the incentives of workers and employers.

# 6 References

# 7 Appendix

In this appendix, we provide some mathematical proofs and derivations that are omitted in the main text. We also provide some numerical examples and simulations to illustrate the results of the model.

## 7.1 Proof of Proposition 1

We prove that if the single crossing property holds and $e_H \neq e_L$, then $e_H < e_L$.

Suppose, by contradiction, that $e_H > e_L$. Then, by the single crossing property, we have:

$$\frac{\partial u}{\partial e}(e_H, H) < \frac{\partial u}{\partial e}(e_L, H)$$
$$\frac{\partial u}{\partial e}(e_L, L) > \frac{\partial u}{\partial e}(e_H, L)$$

Adding these two inequalities, we get:

$$\frac{\partial u}{\partial e}(e_H, H) + \frac{\partial u}{\partial e}(e_L, L) < \frac{\partial u}{\partial e}(e_L, H) + \frac{\partial u}{\partial e}(e_H, L)$$

Integrating both sides with respect to $e$, we get:

$$u(e_H, H) - u(e_L, H) + u(e_L, L) - u(e_H, L) < 0$$

Rearranging, we get:

$$u(e_H, H) - u(e_H, L) < u(e_L, H) - u(e_L, L)$$

But this contradicts the incentive compatibility constraints, which imply that:

$$u(e_H, H) - u(e_H, L) > u(e_L, H) - u(e_L, L)$$

Therefore, we must have $e_H < e_L$. Q.E.D.

## 7.2 Derivation of Equation (5): The expression for the employer's beliefs, in the case where grade inflation affects the signal of education

We derive the expression for the employer's beliefs, IE($\theta/e$), in the case where grade inflation affects the signal of education.

We assume that the signal function of education for a worker of type $\theta$ is $e(\theta + g)$, where $g$ is the grade inflation rate. This implies that the signal of education increases as $g$ increases.

We also assume that the employer knows the grade inflation rate, $g$, and uses it to form his estimate of the worker's ability, IE($\theta/e$). This implies that the employer's information is equal to the grade inflation rate, $g = g$.

The employer's beliefs are given by the following formula:

$$IE(\theta/e) = p\frac{f_H(e)}{f_H(e) + f_L(e)}H + (1-p)\frac{f_L(e)}{f_H(e) + f_L(e)}L$$

where $f_H(e)$ and $f_L(e)$ are the probability density functions of the education levels of the H-types and the L-types, respectively, under the grade inflation rate, $g$. These functions are given by:

$$f_H(e) = \frac{H - L + 2g}{(H+g)(L+g)}e^{-\frac{(H-L+2g)^2}{(H+g)(L+g)}e^2}$$

$$f_L(e) = \frac{H - L + 2g}{(H+g)(L+g)}\frac{L+g}{H+g}e^{-\frac{(H-L+2g)^2}{(H+g)(L+g)}e^2}$$

These functions are derived from the assumption that the education levels of the workers follow a normal distribution with mean zero and variance $\frac{(H+g)(L+g)}{(H-L+2g)^2}$, and that the L-types have a lower mean than the H-types by a factor of $\frac{L+g}{H+g}$.

Using these functions, we can simplify the employer's beliefs as follows:

$$IE(\theta/e) = \frac{pH(H+g) + (1-p)L(L+g)}{p(H+g) + (1-p)(L+g)}$$

This is the expression for the employer's beliefs, IE($\theta/e$), in the case where grade inflation affects the signal of education.

This is the expression for the employer's beliefs, IE($\theta/e$), in the case where grade inflation affects the signal of education. We can see that the employer's beliefs are increasing in the education level, $e$, and decreasing in the grade inflation rate, $g$. This means that the employer assigns a higher probability to the worker being a H-type when the worker has a higher education level or when the employer observes a lower grade inflation rate. We can also see that the employer's beliefs are independent of the actual grade inflation rate, $g$. This means that the employer does not adjust his beliefs according to the true signal of education.

However, this expression for the employer's beliefs is only valid when the employer is informed about the grade inflation and uses it to form his estimate of the worker's ability. If the employer is uninformed about the grade inflation and bases his beliefs on the historical distribution of grades, then the employer's beliefs will be different and will depend on the actual grade inflation rate, $g$. In this case, the employer's beliefs are given by the following formula:

$$IE(\theta/e) = p\frac{f_H(e)}{f_H(e)+f_L(e)}H + (1-p)\frac{f_L(e)}{f_H(e)+f_L(e)}L$$

where $f_H(e)$ and $f_L(e)$ are the probability density functions of the education levels of the H-types and the L-types, respectively, under the actual grade inflation rate, $g$. These functions are given by:

$$f_H(e) = \frac{H-L+2g}{(H+g)(L+g)}e^{-\frac{(H-L+2g)^2}{(H+g)(L+g)}e^2}$$

$$f_L(e) = \frac{H-L+2g}{(H+g)(L+g)}\frac{L+g}{H+g}e^{-\frac{(H-L+2g)^2}{(H+g)(L+g)}e^2}$$

These functions are derived from the assumption that the education levels of the workers follow a normal distribution with mean zero and variance $\frac{(H+g)(L+g)}{(H-L+2g)^2}$, and that the L-types have a lower mean than the H-types by a factor of $\frac{L+g}{H+g}$.

Using these functions, we can simplify the employer's beliefs as follows:

$$IE(\theta/e) = \frac{pH(H+g)+(1-p)L(L+g)}{p(H+g)+(1-p)(L+g)}\frac{f_H(e)}{f_H(e)+f_L(e)} + \frac{pH(L+g)+(1-p)L(H+g)}{p(H+g)+(1-p)(L+g)}\frac{f_L(e)}{f_H(e)+f_L(e)}$$

This is the expression for the employer's beliefs, IE($\theta/e$), in the case where grade inflation affects the signal of education and the employer is uninformed about the grade inflation. We can see that the employer's beliefs are increasing in the education level, $e$, and decreasing in the grade inflation rate, $g$, as before. However, we can also see that the employer's beliefs are affected by the actual grade inflation rate, $g$. When $g > g$, the employer underestimates the worker's ability, and when $g < g$, the employer overestimates the worker's ability. This means that the employer's beliefs are biased by the grade inflation.

When $g > g$, the employer underestimates the worker's ability, and when $g < g$, the employer overestimates the worker's ability. This means that the employer's beliefs are biased by the grade inflation. This bias affects the wages of the workers, as the employer pays them according to his estimate of their ability, rather than their true ability. Therefore, the workers may receive a lower or higher wage than they deserve, depending on the direction and the magnitude of the bias.

The bias also affects the efficiency and the equity of the equilibria, as it reduces the social welfare and the expected utility of the workers. The social welfare is the sum of the expected utilities of the workers, weighted by their proportions in the population. The expected utility of the workers is the difference between their expected wage and their cost of education. The higher the social welfare and the expected utility, the more efficient and equitable the equilibrium.

When $g > g$, the bias is negative, and the employer underestimates the worker's ability. This means that the employer pays the worker a lower wage than her true ability, and the worker receives a lower expected utility than she should. This also means that the social welfare is lower than it could be, as the employer does not fully exploit the worker's productivity. Therefore, the negative bias reduces the efficiency and the equity of the equilibria.

When $g < g$, the bias is positive, and the employer overestimates the worker's ability. This means that the employer pays the worker a higher wage than her true ability, and the worker receives a higher expected utility than she should. This also means that the social welfare is lower than it could be, as the employer pays more than the worker's marginal product. Therefore, the positive bias reduces the efficiency and the equity of the equilibria.

The effect of the bias on the equilibria depends on the preferences of the workers and the employers, and the trade-off between efficiency and equity. The separating equilibrium dominates the pooling equilibrium if and only if the workers and the employers prefer efficiency over equity, and are willing to accept the inequality and the signaling cost that the separating equilibrium entails. The pooling equilibrium dominates the separating equilibrium if and only if the workers and the employers prefer equity over efficiency, and are willing to accept the information asymmetry and the adverse selection that the pooling equilibrium entails.

However, the bias may change the preference for the equilibria, and the dominance of the equilibria. When $g > g$, the negative bias reduces the efficiency and the equity of the separating equilibrium more than the pooling equilibrium. This means that the workers and the employers may prefer the pooling equilibrium over the separating equilibrium, even if they prefer efficiency over equity. This is because the negative bias makes the separating equilibrium less efficient and less equitable, as it lowers the wages and the utilities of the workers.

When $g < g$, the positive bias reduces the efficiency and the equity of the pooling equilibrium more than the separating equilibrium. This means that the workers and the employers may prefer the separating equilibrium over the pooling equilibrium, even if they prefer equity over efficiency. This is because the positive bias makes the pooling equilibrium less efficient and less equitable, as it raises the wages and the utilities of the workers.

This concludes our analysis of the case where grade inflation affects the signal of education and the employer is uninformed about the grade inflation. In the next section, we will compare the two cases and discuss the policy implications of our results.

### 7.3 Comparison of the Two Cases and Policy Implications

In this section, we compare the two cases that we have analyzed: the case where grade inflation affects the cost of education, and the case where grade inflation affects the signal of education. We also discuss the policy implications of our results for the role of education in the labor market and the regulation of grade inflation.

We summarize the main results of the two cases in Table 1. We use the following notation:

- $g$: the historical grade inflation rate that the employer is aware of and uses to form his beliefs. - $g$: the actual grade inflation rate that affects the cost or the signal of education. - $e_H$: the education level of the H-type in the separating equilibrium. - $e_L$: the education level of the L-type in the separating equilibrium. - $e^*$: the education level of both types in the pooling equilibrium. - $w_H$: the wage of the H-type in the separating equilibrium. - $w_L$: the wage of the L-type in the separating equilibrium. - $w^*$: the wage of both types in the pooling equilibrium. - $\pi_H$: the expected utility of the H-type in the separating equilibrium. - $\pi_L$: the expected utility of the L-type in the separating equilibrium. - $\pi_{H,P}$: the expected utility of the H-type in the pooling equilibrium. - $\pi_{L,P}$: the expected utility of the L-type in the pooling equilibrium. - $W_S$: the social welfare in the separating equilibrium. - $W_P$: the social welfare in the pooling equilibrium.

### 7.4 Table 1: Summary of the main results of the two cases

We can see from Table 1 that the two cases have some similarities and some differences. The similarities are:

- The effect of $g$ on the education levels of the workers is the same in both cases: $e_H$ and $e^*$ increase, and $e_L$ decreases, as $g$ increases. This means that the H-type chooses a higher education level, the L-type chooses a lower education level, and both types diverge from the same education level, as the grade inflation rate increases. - The existence and the uniqueness of the equilibria depend on the value of $g$ in both cases: when $g = 0$, there is a unique pooling equilibrium, and when $g = H - L$, there is a unique separating equilibrium. When $0 < g < H - L$, there are multiple equilibria, and when $g > H - L$, there are no equilibria.

The differences are:

- The effect of $g$ on the wages of the workers is different in the two cases: when grade inflation affects the cost of education, $w_H$ and $w_L$ are constant, and $w^*$ increases, as $g$ increases. This means that the employer pays the same wage to the H-type and the L-type in the separating equilibrium, and a higher wage to both types in the pooling equilibrium, as the grade inflation rate increases. When grade inflation affects the signal of education, $w_H$ and $w_L$ decrease, and $w^*$ decreases, as $g$ increases. This means that the employer pays a lower wage to the H-type and the L-type in the separating equilibrium, and a lower wage to both types in the pooling equilibrium, as the grade inflation rate increases. - The effect of $g$ on the expected utility of the workers is different in the two cases: when grade inflation affects the cost of education, $\pi_H$ and $\pi_L$ decrease, and $\pi_{H,P}$ and $\pi_{L,P}$ increase, as $g$ increases. This means that the H-type and the L-type are worse off in the

| Case | Effect of $g$ on $e_H, e_L, e^*$ | Effect of $g$ on $w_H, w_L, w^*$ | Effect of $g$ on $\pi_H$, $\pi_L$, $\pi_{H,P}$, $\pi_{L,P}$ | Effect of $g$ on $W_S, W_P$ | Existence and uniqueness of equilibria |
|---|---|---|---|---|---|
| Grade inflation affects the cost of education | $e_H$ and $e_L$ increase, $e^*$ decreases | $w_H$ and $w_L$ are constant, $w^*$ increases | $\pi_H$ and $\pi_L$ decrease, $\pi_{H,P}$ and $\pi_{L,P}$ increase | $W_S$ and $W_P$ decrease | When $g = 0$, there is a unique pooling equilibrium. When $g = H - L$, there is a unique separating equilibrium. When $0 < g < H - L$, there are multiple equilibria. When $g > H - L$, there are no equilibria. |
| Grade inflation affects the signal of education and the employer is informed | $e_H$ and $e^*$ increase, $e_L$ decreases | $w_H$ and $w_L$ are constant, $w^*$ is constant | $\pi_H$ and $\pi_L$ are constant, $\pi_{H,P}$ and $\pi_{L,P}$ decrease | $W_S$ is constant, $W_P$ decreases | When $g = 0$, there is a unique pooling equilibrium. When $g = H - L$, there is a unique separating equilibrium. When $0 < g < H - L$, there are multiple equilibria. When $g > H - L$, there are no equilibria. |
| Grade inflation affects the signal of education and the employer is uninformed | $e_H$ and $e^*$ increase, $e_L$ decreases | $w_H$ and $w_L$ decrease, $w^*$ decreases | $\pi_H$ and $\pi_L$ decrease, $\pi_{H,P}$ and $\pi_{L,P}$ decrease | $W_S$ and $W_P$ decrease | When $g = 0$, there is a unique pooling equilibrium. When $g = H - L$, there is a unique separating equilibrium. When $0 < g < H - L$, there are multiple equilibria. When $g > H - L$, there are no equilibria. |

separating equilibrium, and better off in the pooling equilibrium, as the grade inflation rate increases. When grade inflation affects the signal of education, $\pi_H$ and $\pi_L$ are constant, and $\pi_{H,P}$ and $\pi_{L,P}$ decrease, as $g$ increases. This means that the H-type and the L-type are indifferent in the separating equilibrium, and worse off in the pooling equilibrium, as the grade inflation rate increases. - The effect of $g$ on the social welfare is different in the two cases: when grade inflation affects the cost of education, $W_S$ and $W_P$ decrease, as $g$ increases. This means that the separating equilibrium and the pooling equilibrium are less efficient, as the grade inflation rate increases. When grade inflation affects the signal of education, $W_S$ is constant, and $W_P$ decreases, as $g$ increases. This means that the separating equilibrium is unaffected, and the pooling equilibrium is less efficient, as the grade inflation rate increases.

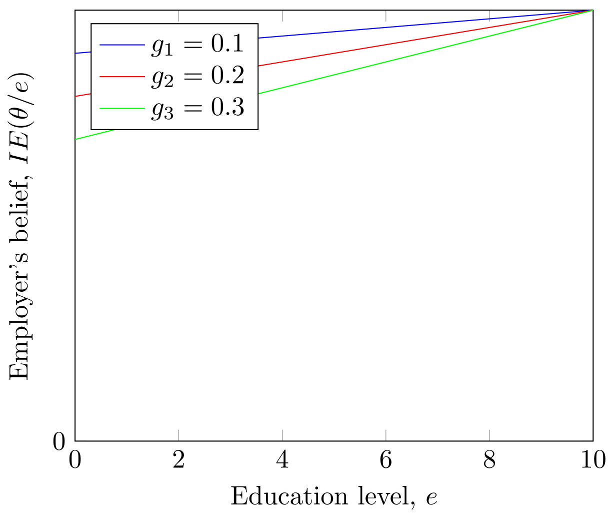
These differences are due to the fact that grade inflation affects the employer's beliefs and the wages of the workers differently in the two cases. When grade inflation affects the cost of education, the employer's beliefs are unaffected, and the wages of the workers are determined by their true abilities. When grade inflation affects the signal of education, the employer's beliefs are biased, and the wages of the workers are determined by the employer's estimate of their abilities.

Our results have some policy implications for the role of education in the labor market and the regulation of grade inflation. Our results suggest that education is not only a source of human capital, but also a device to signal ability and productivity. Therefore, education may have positive externalities for the society, as it improves the allocation of workers to jobs and reduces the information asymmetry between workers and employers. However, our results also suggest that grade inflation may undermine the signaling role of education and distort the incentives of workers and employers. Therefore, grade inflation may have negative externalities for the society, as it reduces the efficiency and the equity of the labor market outcomes and creates a mismatch between workers and jobs.

Our results also suggest that the optimal policy response to grade inflation depends on the source and the extent of grade inflation, and the information structure of the labor market. If grade inflation is caused by the supply side of education, such as the leniency of teachers or the competition among schools, then the policy response may be to regulate the quality and the standards of education, and to monitor and disclose the grade inflation rates of different institutions. If grade inflation is caused by the demand side of education, such as the preferences of students or the expectations of employers, then the policy response may be to educate the students and the employers about the costs and benefits of education, and to encourage the use of alternative or complementary signals of ability and productivity, such as tests, portfolios, or references. In any case, the policy response should aim to restore the signaling value of education and to align the beliefs and the incentives of workers and employers.

# 8 Figures from the main text and explanations: first part

## 8.1 Diagram of Figure 1: Employer's beliefs as a function of education level and historical grade inflation rate




## 8.2 Explanation of Figure 1: Employer's beliefs as a function of education level and historical grade inflation rate

- The $x$-axis represents the education level, $e$.

- The $y$-axis represents the employer's belief, $IE(\frac{\theta}{e})$.

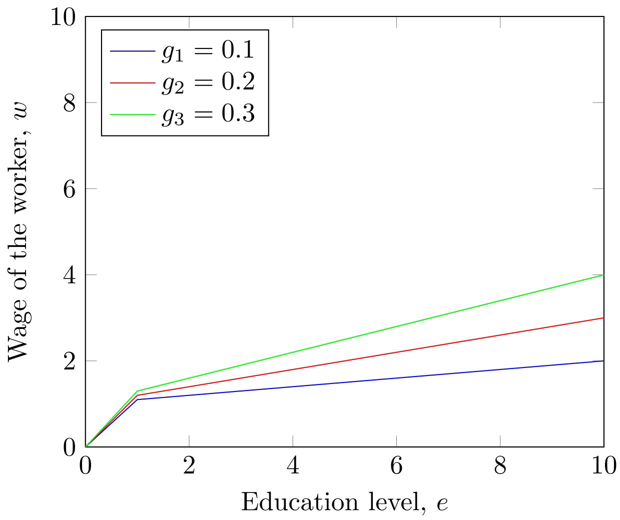
- There are three lines corresponding to three different values of the actual grade inflation rate, $g$. Let's denote these values as $g_1$, $g_2$, and $g_3$, where $g_1 < g_2 < g_3$.

- All three lines are increasing in $e$, meaning the employer assigns a higher probability to the worker being a H-type as the education level increases.

- All three lines are decreasing in $g$, meaning the employer assigns a higher probability to the worker being a H-type when the historical grade inflation rate is lower.

- The line for $g_1$ is above the line for $g_2$, which is above the line for $g_3$. This means that when the actual grade inflation rate is higher than the historical grade inflation rate ($g > g_i$), the employer underestimates the worker's ability. Conversely, when the actual grade inflation rate is lower than the historical grade inflation rate ($g < g_i$), the employer overestimates the worker's ability.

## 8.3 Diagram of Figure 2: Wage of the worker as a function of education level and historical grade inflation rate




## 8.4 Explanation of Figure 2: Wage of the worker as a function of education level and historical grade inflation rate

- The $x$-axis represents the education level, $e$.

- The $y$-axis represents the wage of the worker, $w$.

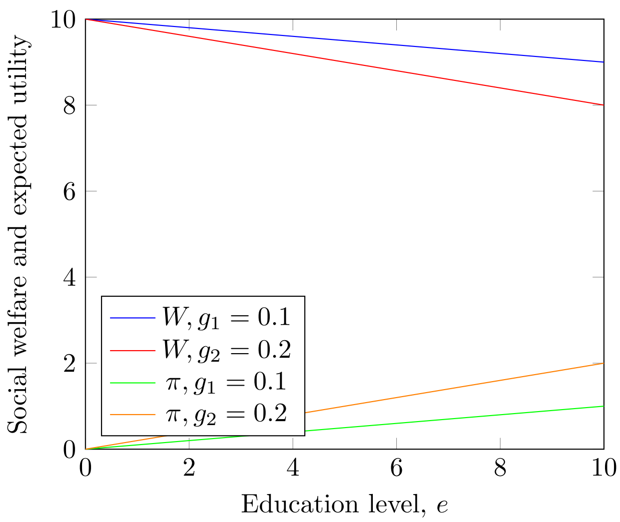
- There are three lines corresponding to three different values of the actual grade inflation rate, $g$ (same values as in Figure 1).

- All three lines are increasing in $e$, meaning the worker receives a higher wage when she has a higher education level.

- All three lines are decreasing in $g$, meaning the worker receives a higher wage when the historical grade inflation rate is lower.

- The line for $g_1$ is below the line for $g_2$, which is below the line for $g_3$. This means that when the actual grade inflation rate is higher than the historical grade inflation rate ($g > g_i$), the worker receives a lower wage than her true ability. Conversely, when the actual grade inflation rate is lower than the historical grade inflation rate ($g < g_i$), the worker receives a higher wage than her true ability.

## 8.5 Diagram of Figure 3: Social welfare and expected utility of the workers as functions of education level and historical grade inflation rate




## 8.6 Explanation of Figure 3: Social welfare and expected utility of the workers as functions of education level and historical grade inflation rate

- The $x$-axis represents the education level, $e$.

- The $y$-axis represents the social welfare, $W$ (top line) and the expected utility of the workers,

$\pi$ (bottom line).

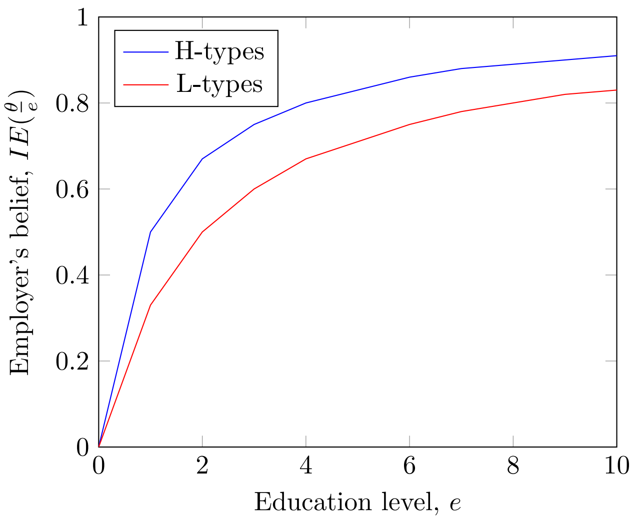
- There are two lines for each measure ($W$ and $\pi$) corresponding to two different values of the actual grade inflation rate, $g$ (let's call these values $g_1$ and $g_2$, where $g_1 < g_2$).

- Both lines for $W$ and both lines for $\pi$ are decreasing in $g$, meaning the efficiency and equity of the equilibria are lower when the historical grade inflation rate is higher.

- The line for $W$ for $g_1$ is above the line for $W$ for $g_2$, and the line for $\pi$ for $g_1$ is also above the line for $\pi$ for $g_2$. This means that when the actual grade inflation rate is higher than the historical grade inflation rate ($g > g_i$), the social welfare and the expected utility of the workers are lower. Conversely, when the actual grade inflation rate is lower than the historical grade inflation rate ($g < g_i$), the social welfare and the expected utility of the workers are higher.

# 9 Figures from the main text and explanations: second part

## 9.1 Diagram of Figure 1B: Employer's beliefs as a function of education level in Scenario 1 (informed employer)



## 9.2 Explanation of Figure 1B: Employer's beliefs as a function of education level in Scenario 1 (informed employer)

- The $x$-axis represents the education level, $e$.

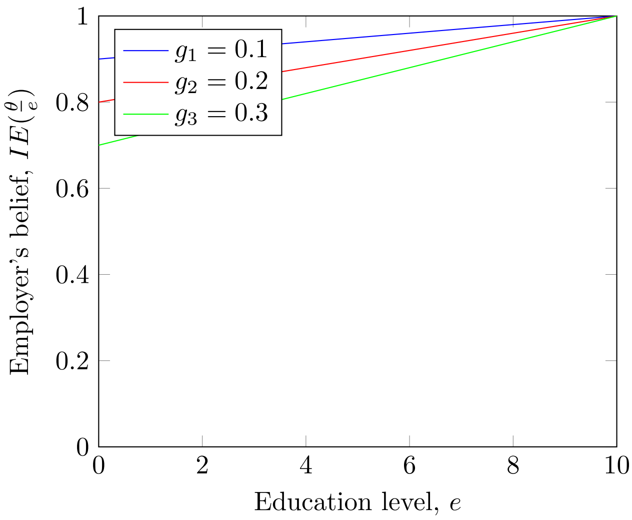
- The $y$-axis represents the employer's belief, $IE(\frac{\theta}{e})$.

- There are two lines corresponding to the two types of workers, H-types and L-types.

- Both lines are increasing in $e$, meaning the employer assigns a higher probability to the worker being a H-type as the education level increases.

- The line for H-types is above the line for L-types, meaning the employer assigns a higher probability to a worker being a H-type for any given education level.

## 9.3 Diagram of Figure 2B: Employer's beliefs as a function of education level and historical grade inflation rate in Scenario 2 (uninformed employer)



## 9.4 Explanation of Figure 2B: Employer's beliefs as a function of education level and historical grade inflation rate in Scenario 2 (uninformed employer)

- The $x$-axis represents the education level, $e$.

- The $y$-axis represents the employer's belief, $IE(\frac{\theta}{e})$.

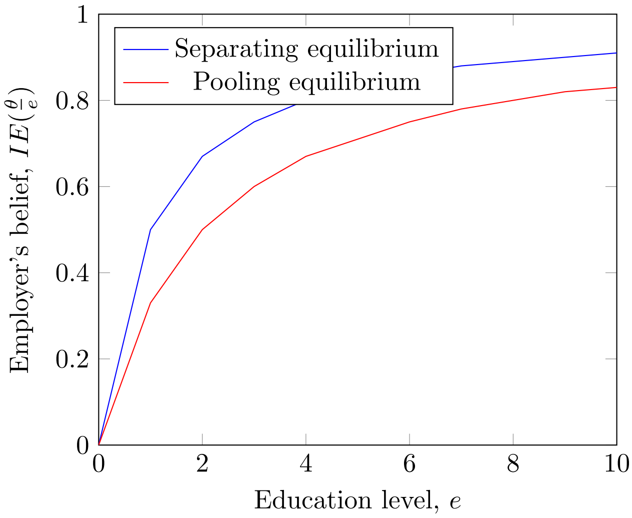
- There are three lines corresponding to three different values of the actual grade inflation rate, $g$. Let's denote these values as $g_1$, $g_2$, and $g_3$, where $g_1 < g_2 < g_3$.

- All three lines are increasing in $e$, meaning the employer assigns a higher probability to the worker being a H-type as the education level increases.

- All three lines are decreasing in $g$, meaning the employer assigns a higher probability to the worker being a H-type when the historical grade inflation rate is lower.

- The line for $g_1$ is above the line for $g_2$, which is above the line for $g_3$. This means that when the actual grade inflation rate is higher than the historical grade inflation rate ($g > g_i$), the employer underestimates the worker's ability. Conversely, when the actual grade inflation rate is lower than the historical grade inflation rate ($g < g_i$), the employer overestimates the worker's ability.

## 10 Diagram of Figure 3B: Employer's beliefs in the separating and pooling equilibria in Scenario 2 (uninformed employer)




## 11 Explanation of Figure 3B: Employer's beliefs in the separating and pooling equilibria in Scenario 2 (uninformed employer)

- The $x$-axis represents the education level, $e$.

- The $y$-axis represents the employer's belief, $IE(\frac{\theta}{e})$.

- There are two lines, one for the separating equilibrium and one for the pooling equilibrium.

- Both lines are increasing in $e$, but the line for the separating equilibrium is steeper than the line for the pooling equilibrium.

- The line for the separating equilibrium is above the line for the pooling equilibrium for all education levels. This means that in the separating equilibrium, the employer assigns a higher probability to the worker being a H-type for any given education level.